Kaji, Hajime; Ohno, Masahiro; Yasukura, Osami
Adjoint varieties and their secant varieties. (English) Zbl 1064.14041
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Summary: The purpose of this article is to show how the graded decomposition of complex simple Lie algebras can be applied to studying adjoint varieties X and their secant varieties $\mathrm{Sec}(X)$. Firstly quadratic equations defining adjoint varieties are explicitly given. Secondly it is shown that $\dim \mathrm{Sec}(X) = 2 \dim X$ for adjoint varieties X in two ways: one is based on Terracini's lemma, and the other is on some explicit description of $\mathrm{Sec}(X)$ in terms of an orbit of the adjoint action. Finally it is shown that the contact loci of the secant variety to its embedded tangent space have dimension two if X is adjoint.

MSC:

14J40 n-folds ($n > 4$)
14M17 Homogeneous spaces and generalizations
14N05 Projective techniques in algebraic geometry

Full Text: DOI

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