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Analysis of an exact fractional step method. (English) Zbl 1130.76394
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Summary: An exact fractional step or projection method for solving the incompressible Navier-Stokes equations is analyzed. The method is applied to both structured and unstructured staggered mesh schemes. There are no splitting errors associated with the method; it satisfies the incompressibility condition to machine precision and reduces the number of unknowns. The exact projection technique is demonstrated on a two-dimensional cavity flow and a multiply connected moving domain with a free surface. Its performance is compared directly to classic fractional step methods and shown to be roughly twice as efficient. Boundary conditions and the relationship of the method to streamfunction-vorticity methods are discussed.

MSC:

76M25 Other numerical methods (fluid mechanics) (MSC2010)
76D05 Navier-Stokes equations for incompressible viscous fluids

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