Cox, S. M.; **Matthews, P. C.**

Exponential time differencing for stiff systems. (English) Zbl 1005.65069

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Summary: We develop a class of numerical methods for stiff systems, based on the method of exponential time differencing. We describe schemes with second- and higher-order accuracy, introduce new Runge-Kutta versions of these schemes, and extend the method to show how it may be applied to systems whose linear part is nondiagonal. We test the method against other common schemes, including integrating factor and linearly implicit methods, and show how it is more accurate in a number of applications. We apply the method to both dissipative and dispersive partial differential equations, after illustrating its behavior using forced ordinary differential equations with stiff linear parts.

MSC:

65L05	Numerical methods for initial value problems
35K30	Initial value problems for higher-order parabolic equations
65M70	Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs
65L06	Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations
34A34	Nonlinear ordinary differential equations and systems, general theory
65L20	Stability and convergence of numerical methods for ordinary differential equations

Keywords:

Runge-Kutta methods; spectral methods; stability; numerical examples; stiff systems; method of exponential time differencing; linearly implicit methods

Software:

Matlab

Full Text: DOI

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