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Differential quadrature solutions of eighth-order boundary-value differential equations. (English) Zbl 1001.65085

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Summary: Special cases of linear eighth-order boundary-value problems have been solved using polynomial splines. However, divergent results were obtained at points adjacent to boundary points. This paper presents an accurate and general approach to solve this class of problems, utilizing the generalized differential quadrature rule (GDQR) proposed recently by the authors. Explicit weighting coefficients are formulated to implement the GDQR for eighth-order differential equations. A mathematically important by-product of this paper is that a new kind of Hermite interpolation functions is derived explicitly for the first time.

Linear and non-linear illustrations are given to show the practical usefulness of the approach developed. Using Fréchet derivatives, non-linear eighth-order problems are also solved for the first time. Numerical results obtained using even only seven sampling points are of excellent accuracy and convergence in an entire domain. The present GDQR has shown clear advantages over the existing methods and demonstrated itself as a general, stable, and accurate numerical method to solve high-order differential equations.

MSC:

65L10 Numerical solution of boundary value problems involving ordinary differential equations
34B05 Linear boundary value problems for ordinary differential equations
65L12 Finite difference and finite volume methods for ordinary differential equations
65L60 Finite element, Rayleigh-Ritz, Galerkin and collocation methods for ordinary differential equations
34B15 Nonlinear boundary value problems for ordinary differential equations
65L20 Stability and convergence of numerical methods for ordinary differential equations

Keywords:

differential quadrature method; finite difference method; eighth-order boundary-value problem; hydrodynamic stability; pseudospectral method; collocation method; numerical results; polynomial splines; convergence

Full Text: DOI

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