Rotman, Joseph J.
Advanced modern algebra. (English) Zbl 0997.00001
Upper Saddle River, NJ: Prentice Hall/Pearson Education. xv, 1012 p., append. 27 p. (2002).

First of all, the book under review is by now the most recent comprehensive, almost encyclopedic text on contemporary algebra at the graduate level. The author has called this book "Advanced Modern Algebra" in homage to the two great classical textbooks "Modern Algebra" by *B. L. van der Waerden* (1930; JFM 56.0138.01), and a "A Survey of Modern Algebra" by *G. Birkhoff* and *S. Mac Lane* (1941; Zbl 0061.04802), both of which presented, at their respective time, the entire subject of algebra at its respective stage of development. There are today major directions in algebra that either did not exist half a century ago, or that were not then recognized to be so important as they turned out to be later on. These new directions involve modern algebraic geometry, computer algebra, homological algebra, representation theory, and others. Although there have appeared other comprehensive textbooks on contemporary algebra, in the meantime, above all the updated version of the classic book by Birkhoff-Mac Lane "Algebra", A. K. Peters (1997; Zbl 0863.00001), *S. Lang*'s widely used text "Algebra" (3rd ed., Addison-Wesley, Reading, 1993; Zbl 0848.13001 and Springer, 2002; Zbl 0984.00001) and, quite recently, *P. Grillet*'s also very modern text "Algebra" (John Wiley & Sons, New York, 1999; Zbl 0939.00002), none of them achieved completely thehighest possible degree of disciplinary broadness or up-to-dateness.

Professor J. Rotman, well-known by his numerous excellent textbooks on various topics in algebra, and driven by his outspoken conviction saying: "Each generation should survey algebra to make it serve the present time", has undertaken the just as ambitious as rewarding task to write another book on contemporary algebra that incorporates the following features simultaneously: appropriate width and diversity, necessary topicality, highest possible self-containedness, effectiveness as a source and reference book, usefulness for teaching and self-instruction, great intelligibility, and consistent rigour.

Based upon his rich experience as a textbook-writer, the author has absolutely succeeded in carrying out this program and reaching his own goals. The outcome is this new standard text on advanced modern algebra, which masterly discusses, on more than a thousand pages and in eleven main chapters, the following fundamental topics in graduate algebra.

Chapters 1, 2 and 3, entitled "Things Past", "Groups I" and "Commutative Rings I", respectively, are to accommodate readers having different backgrounds. These chapters contain the basic facts from undergraduate algebra as they can be found in most introductory textbooks on algebra, and with many proofs merely sketched. This includes some basic set theory, elementary number theory, the first concepts of group theory, polynomials, elementary ring theory, and the basics of linear algebra over fields. The advanced part of the book starts with Chapter 4 which discusses fields, field extensions, algebraic equations, and Galois theory, together with its applications.

Chapter 5, entitled "Groups II';, covers finite abelian groups, the Sylow theorems, the Jordan-Hölder theorem, solvable groups, simple groups, free groups, the Nielsen-Schreier theorem, and presentations of groups.

Chapter 6 comes with the title "Commutative Rings II'; and deals with ideal theory, factorial rings, noetherian rings, higher field theory, Lüroth's theorem, Hilbert's Nullstellensatz and affine algebraic varieties, Gröbner bases, and the Buchberger algorithm in computer algebra.

Chapter 7 turns to categorical algebra and focuses on categories, functors, the category of modules over a ring, free modules, projective modules, injective modules, Grothendieck groups, inverse limits, and direct limits in the category of modules.

Chapter 8, entitled "Algebras", introduces noncommutative rings and algebras, proving Wedderburn's theorem as well as the Wedderburn-Artin theorem classifying semisimple rings. The author also discusses modules over a noncommutative ring, along with tensor products, flat modules, and bilinear forms, covers character theory and Burnside's theorem on solvable finite groups, and finishes this chapter with introducing Frobenius groups.

Chapter 9 is called "Advanced Linear Algebra" and is devoted to several topics in this context, including modules over principal domains, normal forms for matrices, bilinear forms on vector spaces, graded algebras, division algebras, the exterior algebra of a free module, determinants, and the basics on Lie algebras as examples of nonassociative algebras. "Homology" is the title of Chapter 10, which introduces homological methods. Beginning with semidirect products and the extension problem for groups, the author then proceeds with Schreier's solution to the extension problem and the Schur-Zassenhaus lemma, characterizes the functors Ext and Tor after an introduction to homology functors and derived functors, touches upon crossed products, and concludes this chapter with an introduction to spectral sequences.

The final Chapter 11 returns to commutative algebra and is called "Commutative Rings III". Here the author deals with the more advanced topics in the theory of commutative rings, discussing localization, integral extensions, Jacobson rings, Dedekind rings, homological dimensions, Serre's characterization of regular local rings via the concept of global homological dimension, and the Auslander-Buchsbaum theorem on the factoriality of local regular rings.

In an appendix, the author explains, for the convenience of the reader, the set-theoretic aspects of Zorn's lemma. Each section of the book comes with numerous exercises, most of which are equipped with short hints for solution. These exercises are carefully and skillfully selected, which proves once more the great experience and mastery of the author as a teacher of advanced mathematics.

Finally, the text is enhanced by a particularly rich list of references to the (classical and recent) textbook literature in higher algebra. As it appears quite normal for the first edition of a book, there is still quite a number of misprints and minor errors in the text, as well as some proofs that require improvement. The author has carefully compiled many deficiencies of this kind that have been observed or reported, in the meantime, and he has also written up appropriate corrections and improvements of them. The quite extensive list of errata, modified proofs and additional remarks can be found on the author's internet home page `http://www.math.uiuc.edu/~rotman`. This is certainly very useful and, as yet, a solid base for the expected second, corrected edition of this great textbook. Alltogether, it can gladly be stated that the mathematical community has another masterly, brand-new and comprehensive textbook on advanced modern algebra being at its disposal, very much so to the benefit of graduate students, teachers, and researchers using algebraic methods. The methodological and didactic arrangement of the material is very original, interesting, functional, inspiring and efficient. The exposition of the text is consistently lucid, detailed and utmost user-friendly, just as it is customary for this author's books. All this, in its entirety, makes J. Rotman's "Advanced Modern Algebra" a highly welcome enhancement to the existing textbook literature in the field of algebra.

Reviewer: Werner Kleinert (Berlin)

MSC:

00A05	Mathematics in general
12-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to field theory
18-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to category theory
13-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to commutative algebra
16-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to associative rings and algebras
08-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to general algebraic systems

Keywords:

algebraic structures; groups; commutative rings; Galois theory; algebras; categories; homology functors; fields; JFM 56.0138.01; functors; modules; Lie algebras