Ting, L.; **Keller, J. B.**
Weak diffracted shocks near singular rays. (English) Zbl 1024.76021
Methods Appl. Anal. 7, No. 3, 565-576 (2000).

When a shock wave hits a rigid obstacle, it produces reflected and diffracted shocks, and diffracted expansion waves. The reflected shock is shown here to be a straight line segment, and the diffracted wave is shown to be a circular arc. The resulting flow is analyzed near a point P where either the incident shock or the reflected shock meets the diffracted shock. The path of P is called a singular ray, and it is analogous to a ray on a shadow boundary. For self-similar problems, the flow near P is shown to be the solution of a nonlinear elliptic free boundary problem. This flow is regular, in contrast to the singular flow given by linear theory. An analogous problem is found for the interaction of a weak rarefaction wave with a weak shock.

The authors modify previous methods by expanding the solution about the state with the highest characteristic speed, and about the state behind the incident shock. The nonlinear diffracted shock is contained entirely within the sonic circle. There the transonic equation for the pressure is purely elliptic. Thus the authors obtain for the inner solution a nonlinear elliptic free boundary problem instead of a problem of mixed type. Similarly, they expand the solution about the state behind the reflected shock, and after normalization they obtain exactly the same problem.

J. K. Hunter et al. [J. Fluid Mech. 442, 193-205 (2000; Zbl 0995.76042)] showed that the problem he obtained for the transonic small disturbance equation is canonical. This means that it applies in the neighborhood of a singular ray for a general class of hyperbolic systems of partial differential equations in any number of dimensions. As shall be seen for self-similar problems, his problem in the elliptic region with appropriate conditions on the boundary, is the same as the problem discussed. Therefore, this form of the problem is also canonical. In particular, it applies near any point of tangency of a weak shock with a weak diffracted wave, in a self-similar flow.

The authors also formulate a similar problem for the interaction of a weak rarefaction or expansion wave with a weak shock. They show by Hunter's method that this problem is also canonical by Hunter's method. Both canonical problems apply to steady supersonic flow in three dimensions. For simplified forms of the two canonical problems, the authors construct solutions numerically. These solutions could be used in the first steps of iterative procedures to obtain more accurate solutions.

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MSC:

76L05 Shock waves and blast waves in fluid mechanics
35L67 Shocks and singularities for hyperbolic equations

Keywords:

nonlinear elliptic free boundary problem; weak rarefaction wave; transonic small disturbance equation; singular ray; normalization

Full Text: DOI