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A model for the quasi-static growth of brittle fractures: Existence and approximation results. (English) Zbl 1042.74002

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The authors give a precise mathematical formulation of a variational model for the irreversible quasi-static evolution of brittle fractures proposed by *G. A. Francfort* and *J.-J. Marigo* [J. Mech. Phys. Solids 46, No. 8, 1319–1342 (1988; Zbl 0966.74060)], and based on Griffith's theory, where the interplay between bulk and surface energy determines also the crack path. In the two-dimensional case, the authors prove an existence result for the quasi-static evolution of a fracture by using the time-discretization method. To simplify the mathematical description of the model, only linearly elastic homogeneous isotropic materials are considered. The analysis is restricted to the case of anti-plane shear, where the reference configuration is an infinite cylinder $\Omega \times \mathbb{R}$, with $\Omega \subset \mathbb{R}^2$, and the displacement has the special form $(0, 0, u(x_1, x_2))$ for every $(x_1, x_2, y) \in \Omega \times \mathbb{R}$. It is also assumed that the cracks have the form $K \times \mathbb{R}$, where K is a compact set in $\overline{\Omega}$. In this case the notions of bulk energy and surface energy refer to a finite portion of the cylinder determined by two cross-sections separated by a unit distance. The bulk energy is given by $(\mu/2) \int_{\Omega \setminus K} |\nabla u|^2 \, dx$, while the surface energy is given by $k\mathcal{H}^1(K)$, where k is a constant which depends on the toughness of the material, and $\mathcal{H}^1$ is the one-dimensional Hausdorff measure, which coincides with the ordinary length in the case where K is a rectifiable arc. For simplicity the authors take $\mu = 2$ and $k = 1$.

It is shown that the total energy is an absolutely continuous function of time, although the possibility is not excluded that the bulk energy and the surface energy may present some jump discontinuities. This existence result is proved by a time-discretization process, where at each step a global energy minimization is performed, with the constraint that the new crack contains all cracks formed at the previous time steps. This procedure provides an effective way to approximate the continuous time evolution.

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MSC:

74A45 Theories of fracture and damage
74R10 Brittle fracture
74G65 Energy minimization in equilibrium problems in solid mechanics

Keywords:

variational model; anti-plane shear; one-dimensional Hausdorff measure; convergence of minimizers

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