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Stallings foldings and subgroups of free groups. (English) Zbl 1001.20015
J. Algebra 248, No. 2, 608-668 (2002).

In this long paper, the authors study the subgroups of a free group by using the approach of *J. R. Stallings*, who introduced the notion of foldings of graphs [in *Arboreal group theory*, Publ., Math. Sci. Res. Inst. 19, 355-368 (1991; [Zbl 0782.20018](#))]. In their own words “they re-cast in a more combinatorial and computational form the topological approach of J. Stallings to the study of subgroups of free groups”. For this, they give a detailed, self-contained, elementary and comprehensive treatment of the used approach. They also include “complete and independent proofs of most basic facts, a substantial number of explicit examples and a wide assortment of possible applications”. In doing so they reprove many classical well-known, or folklore results about the subgroup structure of free groups. For example they prove the Takahasi-Higman theorem on ascending chains of subgroups of a free group. More precisely: Let $F = F(X)$ be a free group of finite rank. Let $M \geq 1$ be an integer. Then every strictly ascending chain of subgroups of $F(X)$ of rank at most M terminates. Concluding we can say that this paper is a good account of the subgroup structure of free groups in this setting. The reference list contains 48 items.

Reviewer: [Stylianos Andreadakis \(Athens\)](#)

MSC:

[20E05](#) Free nonabelian groups
[57M07](#) Topological methods in group theory
[20E07](#) Subgroup theorems; subgroup growth
[20E15](#) Chains and lattices of subgroups, subnormal subgroups

Cited in **5** Reviews
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Keywords:

[Stallings foldings](#); [subgroups of free groups](#); [ascending chains of subgroups](#)

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