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Representable biresiduated lattices. (English) Zbl 1001.06012

J. Algebra 247, No. 2, 672-691 (2002).

A biresiduated lattice is an algebra $\mathbf{A} = \langle A; \cdot, /, \backslash, \wedge, \vee, 1\rangle$ such that $\langle A; \wedge, \vee\rangle$ is a lattice in which 1 is the maximum element, and / and \ are binary operations which satisfy the following "residuation" properties with respect to the lattice order: $a \cdot c \leq b$ iff $c \leq a\backslash b$; $c \cdot a \leq b$ iff $c \leq b/a$ for all a, b, c in A. The corresponding class is denoted by $\mathcal{B}$. These algebras were first studied by Krull as abstractions of ideal lattices of rings enriched with the monoid operation of ideal multiplication. The objective of this paper is to axiomatize the class of all the biresiduated lattices that may be represented as subalgebras of products of linearly ordered biresiduated lattices (these are called representable and the corresponding class is denoted by $\mathcal{L}$). It is shown that $\mathcal{L}$ is axiomatized, relative to $\mathcal{B}$, by the identity

$$(x\backslash y) \vee ([w \cdot (z\backslash((y\backslash x) \cdot z))]/w) = 1$$

or, equivalently, by

$$(x\backslash y) \vee (w/(w/(((y\backslash x)\backslash z)\backslash z))) = 1.$$

Representable cancellative and complemented biresiduated lattices are also discussed.

Finally, connections with lattice-ordered groups are given.

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MSC:

06F10 Noether lattices
06F15 Ordered groups

Keywords:

axiomatization; biresiduated lattice

Full Text: DOI

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