Chepyzhov, V. V.; **Vishik, M. I.**

Attractors for equations of mathematical physics. (English) Zbl 0986.35001

Colloquium Publications. American Mathematical Society. 49. Providence, RI: American Mathematical Society (AMS). xi, 363 p. (2002).

The book under review is devoted to attractor constructions, and their properties are studied for various non-autonomous differential equations (DEqs) of mathematical physics. Its first part (3 chapters) considers the theory of autonomous evolution equations for which the uniqueness theorem of the relevant Cauchy problem is satisfied. After the theory illustration by examples from ordinary DEqs and the introduction of various functional spaces, a review of the results about the dimension of global attractors for autonomous DEqs is given. Here, upper and lower estimates for the Hausdorff as well as fractal dimensions of attractors are used for concrete equations.

The second part (Chapters IV–IX) of the book deals with attractors of non-autonomous evolution equations. The concept of time symbol of the equation is introduced and families of processes with symbols belonging to some symbol spaces are studied.

On the base of technical Chapter V about the properties of translation compact functions in various function spaces basic non-autonomous evolution equations are studied (2D Navier-Stokes system with time-dependent external force, the reaction-diffusion systems with interaction function and external forces depending on time, the non-autonomous Ginzburg-Landau equation, the non-autonomous dissipative hyperbolic equation). Chapter VII deals with semiprocesses (corresponding to non-autonomous equations with time symbols defined on the semiaxis $\mathbb{R}_+$) and their attractors. The behaviour of solutions as $t \to +\infty$ are studied and uniform attractors existence theorems are proved. The importance for the description of the uniform global attractors general structure notion of the kernels for non-autonomous evolution equations is introduced in Chapter VIII. Certain weak invariance and attracting properties of the kernel of a given process are established and the fractal dimension of a kernel section is studied. It is finite and has the uniform upper bound similar to the relevant autonomous case. In Chapter IX upper estimates are proved for the Kolmogorov ε-entropy of uniform attractors for non-autonomous evolution equations with translation compact symbols. But also some classes of symbol spaces with infinite fractal dimension are considered, for which upper bounds for their ε-entropy are deduced.

In the third part (Chapters X–XVIII) trajectory attractors are studied for autonomous and non-autonomous equations of mathematical physics. Here it is not assumed the unique solvability of the relevant Cauchy problems. At first the method of trajectory attractors is explained on autonomous ordinary DEqs. Then the global attractor existence theorem is proved for an abstract semigroup acting in a general Hausdorff topological space (Chapter XI). This result necessity is explained by the fact that in applications one deals with translation semigroups acting in various topological spaces with local weak topology which is not metrizable. Chapter XII presents the theory of trajectory attractors for absract autonomous evolution equations without unique solvability assumption of the relevant Cauchy problems. A trajectory attractor existence theorem is proved and its structure is described in terms of kernels. In Chapter XIII trajectory attractors and global attractors are studied for autonomous equations of mathematical physics (3D Navier-Stokes system and the dissipative hyperbolic equation with nonlinear term of high polynomial growth), for which the uniqueness theorem is not yet proved. For these equations containing small parameters the perturbations of trajectory and global attractors are studied. Chapter XIV is devoted to the construction of a uniform trajectory attractor for a non-autonomous evolution equation written in abstract operatorial form. Its results are applied in Chapter XV to the non-autonomous 2D and 3D Navier-Stokes systems, reaction-diffusion equations, Ginzburg-Landau equation, and dissipative hyperbolic equations. In the concluding three chapters the authors study the approximation of trajectory global attractors by those notions for the relevant Galerkin systems, perturbations of trajectory attractors as well as global attractors for non-autonomous partial DEqs, and also the averaging of attractors for evolution equations with rapidly oscillating terms.

The reviewed book elucidates the contemporary state of the qualitative theory of partial differential equations and therefore will be very useful for both theoretical and applied mathematicians.

Reviewer: Boris V.Loginov (Ulyanovsk)

MSC:

35-02 Research exposition (monographs, survey articles) pertaining to partial differential equations
37-02 Research exposition (monographs, survey articles) pertaining to dynamical systems and ergodic theory
35B41 Attractors
35K90 Abstract parabolic equations
35B40 Asymptotic behavior of solutions to PDEs
37C70 Attractors and repellers of smooth dynamical systems and their topological structure
37L30 Infinite-dimensional dissipative dynamical systems–attractors and their dimensions, Lyapunov exponents
35Q30 Navier-Stokes equations
35L70 Second-order nonlinear hyperbolic equations

Keywords:

non-autonomous evolution equations; Kolmogorov ε-entropy of attractors; trajectory attractors; semiprocesses; 3D Navier-Stokes system