Parshin, A. N.

Integrable systems and local fields. (English) Zbl 1014.14015

Commun. Algebra 29, No. 9, 4157-4181 (2001).

The well-known Krichever corresdondence, linking algebro-geometric data with the theory of infinite-dimensional Grassmannians and algebras of differential operators, has been a subject of vast research during the past 25 years. This construction has been successfully used in the theory of integrable Hamiltonian systems, particularly in the theory of the KP and KdV equations, and it has been an essential ingredient for the final proofs of some longstanding conjectures such as the Schottky problem and the Novikov conjecture. Also, various attempts have been made, over the past two decades, to generalize Krichever's construction in different directions.

The paper under review is devoted to a purely algebraic approach to the so-called Krichever map. Just assuming an arbitrary groundfield k of characteristic zero, the author describes a general connection between the KP hierarchy (in Lax form) and the vector fields on infinite Grassmannians. This is not only more general, but also much simpler and much more systematic and transparent than the previously known algebraic-analytic constructions. Then, using his theory of adelic complexes [cf. *T. Fimmel* and *A. N. Parshin*, "Introduction to higher adelic theory" (book to appear)], the author establishes the general Krichever correspondence in dimension one, before extending it to algebraic surfaces in the following sections of the paper.

The Krichever correspondence appears here as a quasi-isomorphism between certain adelic complexes, and it is precisely this form that suggests the possibility of further generalizations to higher-dimensional varieties. An approach to such a further generalization has been proposed, in the meantime, by *D. V. Osipov* in his recent paper "Krichever correspondence for algebraic varieties" [Izv. Math. 65, 941-975 (2001); translation from Izv. Ross. Akad. Nauk, Ser. Mat. 65, 91-128 (2001; Zbl 1068.14053)].

Altogether, this paper reveals the apparently very general character of the Krichever correspondence as well as its crucial significance and ubiquity. Apart from establishing the Krichever correspondence in dimension two, the purely algebraic and far-reaching methodical approach represents the pioneering character of this work.

Reviewer: Werner Kleinert (Berlin)

MSC:

14H70	Relationships between algebraic curves and integrable systems
37K20	Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with algebraic geometry, complex analysis, and special functions
13N10	Commutative rings of differential operators and their modules
14G20	Local ground fields in algebraic geometry

Keywords:

Krichever corresdondence; KP hierarchy; infinite Grassmannians

Full Text: DOI arXiv

References:

[1] Álvarez Vásquez A., The Algebraic Formalism of Soliton Equations over Arbitrary Base Fields · Zbl 0995.14021

[2] Arbarello E., Comm. Math. Phys. 117 pp 1– (1988) · Zbl 0647.17010 · doi:10.1007/BF01228409

[3] Beilinson A. A., Funct. Analysis and its Appl. 14 pp 34– (1980) · Zbl 0509.14018 · doi:10.1007/BF01078412

[4] Beilinson A. A., Comm. Math. Physics 118 pp 651– (1988) · Zbl 0665.17010 · doi:10.1007/BF01221114

[5] Ben-Zvi D., Spectral Curves, Opers and Integrable Systems · Zbl 1113.14301 · doi:10.1007/s10240-001-8193-1

[6] Bourbaki N., Algèbre Commutative (1961)

[7] Fimmel T., Introduction to the Higher Adelic Theory
[8] Grothendieck A., Public. Mathem. IHES pp 8– (1961)
[9] Hartshorne R., Algebraic Geometry (1977) · doi:10.1007/978-1-4757-3849-0
[10] Huber A., Abh. Math. Sem. Univ. Hamburg 61 pp 249– (1991) · Zbl 0763.14006 · doi:10.1007/BF02950770
[11] Kashiwara M., Algebraic Analysis, Geometry and Number Theory pp 161– (1989)
[12] Krichever I. M., Russian Math. Surveys 32 pp 185– (1977) · Zbl 0386.35002 · doi:10.1070/RM1977v032n06ABEH003862
[13] Li Y., Category of Morphisms of Algebraic Curves and a Characterization of Prym Varieties
[14] Matsumura H., Commutative Ring Theory (1986) · Zbl 0603.13001
[15] Mulase M., Int. J. Math. 1 pp 293– (1990) · Zbl 0723.14010 · doi:10.1142/S0129167X90000174
[16] Osipov D. V., Izvestija RAN, Ser. Math. 65 (2001)
[17] Parshin A. N., Izvestija AN SSSR, Ser. Math. 40 pp 736– (1976)
[18] Parshin A. N., Algebra and Number Theory pp 165– (1994)
[19] Parshin A. N., Selected Works pp 535– (1998)
[20] Parshin A. N., Proc. Steklov Math. Institute 224 pp 266– (1999)
[21] Pressley A., Loop groups (1986) · Zbl 0618.22011
[22] Quandt I., On a Relative Version of the Krichever Correspondence · Zbl 0907.14004
[23] Sato M., Lecture Notes in Num. Appl. Anal. 5 pp 259– (1982)
[24] Segal G., Publ. Math. IHES 80 pp 301– (1985)