Verch, Rainer

A spin-statistics theorem for quantum fields on curved spacetime manifolds in a generally covariant framework. (English) Zbl 1086.81067

Commun. Math. Phys. 223, No. 2, 261-288 (2001).

The paper under review proves a spin-statistics theorem for locally generally covariant quantum field theories over four-dimensional, globally hyperbolic spacetimes. The proof is based on a deformation argument for the spacetime as well as on the elaborate notion of *generally covariant quantum field theory* over the set $\mathcal{G}$ of all four-dimensional, globally hyperbolic spacetimes with spin-structure. The latter definition has several ingredients:

(i) a family of quantum fields of a fixed spinor or tensor type labeled by the spacetimes in $\mathcal{G}$,

(ii) a natural way of identifying the von Neumann algebras generated by these fields localized in open, relatively compact sets of suitable subregions of different spacetimes in $\mathcal{G}$,

(iii) a causal dynamical law and (iv) the usual properties of the corresponding quantum field theory when considered over Minkowski space, which is an element of $\mathcal{G}$.

This framework is already explained in the first section of the paper which also contains an informative review of the connection between spin and statistics in the different physical and mathematical situations where it has been stated.

The following two sections are devoted to more geometrical aspects: spacetimes with spin structure and corresponding local morphisms are introduced, the notion of globally hyperbolic sub-spacetime as well as the crucial deformation argument are stated (cf. Lemma 2.1 and Appendix A), the latter being an extension of the methods considered by *S. A. Fulling, F. J. Narcowich* and *R. M. Wald* in [Ann. Phys. 136, 243–272 (1981; Zbl 0495.35054)].

Roughly speaking this useful procedure allows to deform (preserving global hyperbolicity) some regions of spacetime in such a way that they become isomorphic to Minkowski spacetime leaving the rest unchanged. Once this geometrical context is established, the notion of a generally covariant quantum field theory over $\mathcal{G}$ (which is defined precisely in Section 4) allows to prove the connection between spin and statistics for these theories using among other things the corresponding theorem which is true in Minkowski space. Finally, the Klein-Gordon, the Dirac and the Proca quantum fields over globally hyperbolic spacetimes and specified in a Hadamard state are given as examples of generally covariant quantum field theories.

From a mathematical point of view the paper is written carefully and uses an elaborate geometrical and functional-analytical language. The author gives also in several useful and precise remarks the relation of his definitions and methods to similar ones already existing in the literature; e.g. for the connection between spin and statistics cf. *L. Parker* and *Y. Wang* [Phys. Rev. D 39, 3596–3605 (1989)] or *R. Wald* [Ann. Phys. 118, 490–510 (1979)] and for the notion of general covariance see also *J. Dimock* in [Commun. Math. Phys. 77, 219–228 (1980; Zbl 0455.58030)] and [Trans. Am. Math. Soc. 269, 133–147 (1982; Zbl 0518.58018)], *B.S. Kay* in [Proceedings of the Accademia Nazionale dei Lincei (July 1-6, 1996), Cambridge, MA: International Press. 578–588 (1997; Zbl 0918.53027)] or *U. Bannier* in [Commun. Math. Phys. 118, No. 1, 163–170 (1988; Zbl 0655.46058)].

Reviewer: Fernando Lledó (Aachen)

MSC:

81T20 Quantum field theory on curved space or space-time backgrounds
46L60 Applications of selfadjoint operator algebras to physics
81T05 Axiomatic quantum field theory; operator algebras

Keywords:

spin-statistics theorem; quantum field theory on curved spacetime; generally covariant quantum fields

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