McMullen, Curtis T.
Local connectivity, Kleinian groups and geodesics on the blowup of the torus. (English)
Zbl 1061.37025
Invent. Math. 146, No. 1, 35-91 (2001).

Let $N = \mathbb{H}^3/\Gamma$ be a complete hyperbolic 3-manifold with a free fundamental group $\pi_1(N) \cong \Gamma \cong \langle A, B \rangle$, such that the commutator $[A, B]$ is parabolic. The limit set Λ of N is the locus of chaotic dynamics for the action of $\pi_1(N)$ on $S_\infty^2 = \partial\mathbb{H}^3$. The author shows that the topological dynamical system $(\Lambda, \pi_1(N))$ is always a quotient of the standard action of a surface group on a circle. Let Σ be a compact surface of genus one with a single boundary component. Its interior Σ^0 can be endowed with a complete hyperbolic metric of finite volume, providing a natural action of $\pi_1(\Sigma)$ on the circle $S_\infty^1 = \partial\widetilde{\Sigma}^0 \cong \partial\mathbb{H}$. There is a homotopy equivalence or marking $f : \Sigma \to N$, sending $\partial\Sigma$ to a cusp of N. Let $H(\Sigma)$ denote the set of all such marked hyperbolic 3-manifolds.

Some results are the following theorem and corollary: Theorem 1.1. For any $N \in H(\Sigma)$ there is a natural, continuous, surjective map $F : S_\infty^1 \to \Lambda \subset S_\infty^2$, respecting the action of $\pi_1(\Sigma)$. Corollary 1.2. The limit set of any $N \in H(\Sigma)$ is locally connected. Conjecture 1.3. For any hyperbolic 3-manifold N with finitely generated fundamental group, there exists a continuous, $\pi_1(N)$-equivariant map $F : \partial\pi^1(N) \to \Lambda \subset S_\infty^2$. Theorem 1.1 is a special case of this conjecture.

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MSC:

37F30	Quasiconformal methods and Teichmüller theory, etc. (dynamical systems) (MSC2010)
37C85	Dynamics induced by group actions other than $\mathbb{Z}$ and $\mathbb{R}$, and $\mathbb{C}$
57M50	General geometric structures on low-dimensional manifolds
30F40	Kleinian groups (aspects of compact Riemann surfaces and uniformization)
30F60	Teichmüller theory for Riemann surfaces

Cited in **2** Reviews
Cited in **21** Documents

Keywords:

complete hyperbolic 3-manifold with free fundamental group; surfaces group; complete hyperbolic metric of finite volume

Full Text: DOI