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Packing congruent bricks into a cube. (English) Zbl 1008.52017
J. Geom. Graph. 5, No. 1, 1-11 (2001).

Summary: *L. Lovász* raised the problem whether 27 congruent bricks of edge lengths a, b, c ($0 < a < b < c$, $a + b + c = s$) can be packed into a cube of edge length s without overlaps so that the arrangement is universal, in other word it should be independent of the choice of a, b and c. If that were possible, we could obtain a geometric proof of the inequality $\frac{1}{3}(a + b + c) \geq \sqrt[3]{abc}$ between the arithmetic and geometric means of three positive numbers. (This would be an analogous method to the well-known proof of the inequality $\frac{1}{2}(a + b) \geq \sqrt{ab}$, $a, b > 0$, concerning the packing of four rectangles of edge lengths a, b into a square of edge length $a + b$.)

Hence, fundamentally, this is a special packing problem: some bricks having fixed volume must be put into a container of given volume. From the combinatorial point of view, similar container problems were investigated by *D. Jennings* [J. Comb. Theory, Ser. A 68, No. 2, 465-469 (1994; Zbl 0808.05033) and Discrete Math. 138, No. 1-3, 293-300 (1995; Zbl 0836.52005)]. The first author has found a possible universal arrangement, and someone else has found an additional one which has proved to be different under the symmetries of the cube. In the present paper we introduce an algorithm for finding all the different universal arrangements. As a result we obtain 21 possibilities (listed in Section 4) by the corresponding computer program. Our method seems to be suitable for solving the analogous problem in higher dimensions.

MSC:

52C17 Packing and covering in n dimensions (aspects of discrete geometry)

Keywords:

packing problem; classification problem

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