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Ground state entropy of the Potts antiferromagnet on triangular lattice strips. (English)

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Summary: The authors present exact calculations of the zero-temperature partition function (chromatic polynomial) P for the q -state Potts antiferromagnet on triangular lattice strips of arbitrarily great length L_x vertices and of width L_y vertices and, in the $L_x \rightarrow \infty$ limit, the exponent of the ground state entropy $W = e^{S_0/k_B}$. The strips considered, with their boundary conditions (BC), are (a) $(FBC_y, PBC_x) =$ cyclic for $L_y = 3, 4$, (b) $(FBC_y, TPBC_x) =$ Möbius, $L_y = 3$, (c) $(PBC_y, PBQ_x) =$ toroidal, $L_y = 3$, (d) $(PBC_y, TPBC_x) =$ Klein bottle, $L_y = 3$, (e) $(PBC_y, FBC_x) =$ cylindrical, $L_y = 5, 6$, and (f) $(FBC_y, FBC_x) =$ free, $L_y = 5$, where F, P , and TP denote free, periodic, and twisted periodic. Several interesting features are found, including the presence of terms in P proportional to $\cos(2\pi L_x/3)$ for case (c). The continuous locus of points \mathcal{B} where W is nonanalytic in the q plane is discussed for each case and a comparative discussion is given of the respective loci \mathcal{B} for families with different boundary conditions. Numerical values of W are given for infinite-length strips of various widths and are shown to approach values for the 2D lattice rapidly. A remark is also made concerning a zero-free region for chromatic zeros. Some results are given for strips of other lattices.

MSC:

82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics Cited in 1 Document

Keywords:

q-state Potts antiferromagnet; exact calculations; zero-temperature partition function; triangular lattice strips

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References:

- [1] Potts, R.B., Math. proc. Cambridge philos. soc., 48, 106, (1952)
- [2] Wu, F.Y., Rev. mod. phys., 54, 235, (1982)
- [3] Read, R.C., J. combin. theory, 4, 52, (1968)
- [4] W. T. Tutte, \textit{in} Lecture Notes in Math., Vol. 411, p. 243, Springer-Verlag, New York/Berlin, 1974.
- [5] Read, R.C.; Tutte, W.T., Selected topics in graph theory, (1988), Academic Press New York
- [6] Biggs, N.L., Algebraic graph theory, (1993), Cambridge Univ. Press Cambridge · Zbl 0501.05039
- [7] Shrock, R.; Tsai, S.-H., Phys. rev. E, 55, 5165, (1997)
- [8] A brief presentation of some of these results was given by R.S. in a review talk at the Rutgers Statistical Mechanics Meeting, December 1999.
- [9] Beraha, S.; Kahane, J.; Weiss, N., J. combin. theory B, 27, 1, (1979)
- [10] S. Beraha, J. Kahane, and, N. Weiss, 28, 1980, 52.
- [11] Biggs, N.L.; Damerell, R.; Sands, D., J. combin. theory B, 12, 123, (1972)
- [12] D. Sands, Ph.D. Thesis, University of London, 1972, (unpublished; copy courtesy of Professor N. L. Biggs).
- [13] Biggs, N.L.; Meredith, G., J. combin. theory B, 20, 5, (1976)
- [14] Biggs, N.L., Bull. London math. soc., 9, 54, (1977)
- [15] R. C. Read, \textit{in}, Proc. 5th Caribbean Conf. on Combin. and Computing, 1988.
- [16] Read, R.C.; Royle, G.F., Graph theory, combinatorics, and applications, (1991), Wiley New York, p. 1009
- [17] Shrock, R.; Tsai, S.-H., Phys. rev. E, 60, 12, (1999)
- [18] Shrock, R.; Tsai, S.-H., Physica A, 275, 429, (2000)

- [19] Shrock, R., Phys. lett. A, 261, 57, (1999)
- [20] S.-C. Chang, and, R. Shrock, Physica A, in press; cond-mat/0004161.
- [21] N. L. Biggs, LSE report LSE-CDAM-99-03, May 1999.
- [22] Shrock, R.; Tsai, S.-H., J. phys. A lett., 32, L195, (1999)
- [23] Shrock, R.; Tsai, S.-H., J. phys. A, 32, 5053, (1999)
- [24] R. Shrock, Discrete Math, in press, cond-mat/9908387;
- [25] Shrock, R., Physica A, 281, 221, (2000)
- [26] Shrock, R., Physica A, 281, 221, (2000)
- [27] Roček, M.; Shrock, R.; Tsai, S.-H., Physica A, 252, 505, (1998)
- [28] Roček, M.; Shrock, R.; Tsai, S.-H., Physica A, 259, 367, (1998)
- [29] Shrock, R.; Tsai, S.-H., Physica A, 259, 315, (1998)
- [30] Biggs, N.L.; Shrock, R., J. phys. A, 32, L489, (1999)
- [31] Baxter, R.J., J. phys. A, 20, 5241, (1987)
- [32] Baxter, R.J., J. phys. A, 19, 2821, (1986)
- [33] Shrock, R.; Tsai, S.-H., Phys. rev. E, 58, 4332, (1998)
- [34] H. Klüpfel, and, R. Shrock, unpublished;
- [35] Klüpfel, H., Stony brook thesis, (July 1999)
- [36] Chang, S.-C.; Shrock, R., Physica A, 286, 189, (2000)
- [37] Shrock, R.; Tsai, S.-H., Phys. rev. E, 56, 3935, (1997)
- [38] Shrock, R.; Tsai, S.-H., J. phys. A, 31, 9641, (1998)
- [39] Shrock, R.; Tsai, S.-H., Physica A, 265, 186, (1999)
- [40] A. Sokal, Combin. Prob. Comput, in press; cond-mat/9904146.
- [41] S.-C. Chang, and, R. Shrock, Physica A, in press; cond-mat/0005232.
- [42] Matveev, V.; Shrock, R., J. phys. A, 28, 5235, (1995)
- [43] Shrock, R.; Tsai, S.-H., Phys. rev. E, 55, 6791, (1997)
- [44] Shrock, R.; Tsai, S.-H., Phys. rev. E, 56, 4111, (1997)
- [45] Wannier, G., Phys. rev., 79, 357, (1950)
- [46] Stephenson, J., J. math. phys., 5, 1009, (1964)
- [47] Jackson, B., Combin. prob. comput., 2, 325, (1993)
- [48] Thomassen, C., Combin. prob. comput., 6, 497, (1997)
- [49] Brown, J., J. combin. theory B, 72, 251, (1998)
- [50] Shrock, R.; Tsai, S.-H., Phys. rev. E, 56, 1342, (1997)

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