Yudovich, V. I.

On the loss of smoothness of the solutions of the Euler equations and the inherent instability of flows of an ideal fluid. (English) Zbl 0982.76014

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Summary: We study certain classes of flows of ideal incompressible fluid which with time gradually lose their smoothness. The loss of smoothness is expressed as infinite growth of vorticity as $t \to \infty$ for three-dimensional flows, and as an increase in the gradient of vorticity for plane and axisymmetric flows. Examples of such flows in the plane and axisymmetric cases are flows with a rectilinear streamline; this can be established using a special local Lyapunov function. Incompressible flows of a dusty medium are another example (it turns out that collapse is impossible for such flows, but the vorticity and the rate of deformation, as a rule, grow unboundedly). Other examples can be constructed by composition of shear flows. Here we show that in the vorticity metric almost all stationary plane flows are unstable with respect to three-dimensional disturbances, and in the vorticity gradient metric plane and axisymmetric flows with a rectilinear streamline are unstable.

MSC:

76B03 Existence, uniqueness, and regularity theory for incompressible inviscid fluids
76E99 Hydrodynamic stability
35Q35 PDEs in connection with fluid mechanics

Keywords:

flow instability; loss of smoothness; ideal incompressible fluid; axisymmetric flows; local Lyapunov function; vorticity metric; plane flows; three-dimensional disturbances; vorticity gradient metric; flows with rectilinear streamline

Full Text: DOI

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