Chang, S.-C.; **Shrock, R.**

Exact Potts model partition functions on strips of the honeycomb lattice. (English)

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Summary: We present exact calculations of the partition function of the q-state Potts model on (i) open, (ii) cyclic, and (iii) Möbius strips of the honeycomb (brick) lattice of width $L_y = 2$ and arbitrarily great length. In the infinite-length limit the thermodynamic properties are discussed. The continuous locus of singularities of the free energy is determined in the q plane for fixed temperature and in the complex temperature plane for fixed q values. We also give exact calculations of the zero-temperature partition function (chromatic polynomial) and $W(q)$, the exponent of the ground-state entropy, for the Potts antiferromagnet for honeycomb strips of type (iv) $L_y = 3$, cyclic, (v) $L_y = 3$, Möbius, (vi) $L_y = 4$, cylindrical, and (vii) $L_y = 4$, open. In the infinite-length limit we calculate $W(q)$ and determine the continuous locus of points where it is nonanalytic. We show that our exact calculation of the entropy for the $L_y = 4$ strip with cylindrical boundary conditions provides an extremely accurate approximation, to a few parts in 10^5 for moderate q values, to the entropy for the full 2D honeycomb lattice (where the latter is determined by Monte Carlo measurements since no exact analytic form is known).

MSC:

82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics

Keywords:

infinite-length limit; thermodynamic properties; zero temperature

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.