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Augmentation modules for affine groups. (English) Zbl 1005.20005
Math. Proc. Camb. Philos. Soc. 130, No. 2, 287-294 (2001).

In 1991, *D. M. Evans* and the reviewer [Q. J. Math., Oxf. II. Ser. 42, No. 165, 15-26 (1991; Zbl 0719.20002)] considered the possible structure of permutation modules defined for some infinite permutation groups. As a result of the results in that paper the reviewer made a conjecture which this paper proves. Let F be a field and let G be the n -dimensional affine group $\text{AGL}(n, F)$. Consider the permutation representation on the translation subgroup. Let k be a field and consider the permutation module over k for this action. This module always has at least one proper non-trivial kG -submodule, the augmentation module. For infinite fields the conjecture was that if k and F had different characteristics then the augmentation module is the only proper non-trivial kG -submodule. The case where F was the rational field was dealt with by *Camina* and *Evans* and when F is a finite extension of the rational field by *D. R. Farkas* and *R. L. Snider* [ibid. 45, No. 177, 29-42 (1994; Zbl 0802.20006)]. The situation for finite fields is well known and as a consequence it is easy to see that the conjecture holds for locally finite fields.

The proof is essentially ring theoretic. They consider the group algebra KA where A is the translation subgroup with the action of Q , where $Q = G/A$. Then the kG -submodules correspond to Q -invariant ideals. This enables the authors to use knowledge of the primes in the ring to be used. A number of recent papers have used these results [*D. S. Passman*, Trans. Am. Math. Soc. 354, No. 8, 3379-3408 (2002; Zbl 0998.16017); *D. S. Passman* and *A. E. Zalesskij*, Proc. Am. Math. Soc. 130, No. 4, 939-949 (2002; Zbl 0992.16021); *J. M. Osterburg*, *D. S. Passman* and *A. E. Zalesskij*, ibid. 130, No. 4, 951-957 (2002; Zbl 0992.16022)].

Reviewer: [Alan R. Camina \(Norwich\)](#)

MSC:

- 20C07 Group rings of infinite groups and their modules (group-theoretic aspects)
- 20B07 General theory for infinite permutation groups
- 16S34 Group rings
- 16D25 Ideals in associative algebras

Cited in **2** Reviews
Cited in **8** Documents

Keywords:

permutation modules; infinite permutation groups; affine groups; permutation representations; translation subgroups; augmentation modules; locally finite fields; group algebras

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