Mamedov, M. A.; **Pehlivan, S.**
Statistical convergence of optimal paths. (English) Zbl 0964.40001
Math. Jap. 52, No. 1, 51-55 (2000).

The concept of statistical convergence was introduced for number sequences by *H. Fast* [Colloq. Math. 2, 241-244 (1951; Zbl 0044.33605)] and it can be easily extended to sequences in a normed linear space E with the norm $\|\|$. A sequence $(x_n)_1^\infty$ of elements of E is called a statistically convergent sequence to $\xi \in E$ provided that $d(A(\varepsilon)) = 0$, where

$$A(\varepsilon) = \{n : \|x_n - \xi\| \geq \varepsilon\} \quad \text{and} \quad d\big(A(\varepsilon)\big) = \lim_{n\to\infty} \frac{1}{n} \text{card}\big(\{1, 2, \ldots, n\} \cap A(\varepsilon)\big).$$

J. A. Fridy introduced the concepts of statistical limit points and statistical cluster points of sequences; cf. *J. A. Fridy* [Proc. Am. Math. Soc. 118, No. 4, 1187-1192 (1993; Zbl 0776.40001)]. Consider the system: $x_{k+1} = f(x_k, u_k)$, $x_1 = \xi^{(0)} \in R^m$, $f(x, u) : R^m \times R^r \to R^m$ a continuous function, $u \in U \subset R^r$, U a compact set. Then the sequences (x_k), (u_k) are called a path and a control, respectively. In the paper the asymptotic behaviour of optimal paths and optimal controls are studied. It is shown that under some conditions all optimal paths are statistically convergent to the unique stationary point which is a statistical limit point and also a cluster point for all optimal paths.

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MSC:

40A05 Convergence and divergence of series and sequences
49J24 Optimal control problems with differential inclusions (existence) (MSC2000)

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