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Analytical solutions describing the phase separation driven by a free energy functional containing a long-range interaction term. (English) Zbl 0970.35151

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Summary: We are primarily concerned with the variational problem with long-range interaction. This functional represents the Gibbs free energy of the microphase separation of diblock copolymer melts. The critical points of this variational problem can be regarded as the thermodynamic equilibrium state of the phase separation phenomenon. Experimentally it is well known in the diblock copolymer problem that the final equilibrium state prefers periodic structures such as lamellar, column, spherical, double-diamond geometries and so on. We are interested in the characterization of the periodic structure of the global minimizer of the functional (corresponding to the strong segregation limit). In this paper we completely determine the principal part of the asymptotic expansion of the period with respect to ε (interfacial thickness), namely, we estimate the higher-order error term of the period with respect to ε in a mathematically rigorous way in one-space dimension.

Moreover, we decide clearly the dependency of the constant of proportion upon the ratio of the length of two homopolymers and upon the quench depth. In the last section, we study the time evolution of the system. We first study the linear stability of spatially homogeneous steady state and derive the most unstable wavelength, if it is unstable. This is related to spinodal decomposition. Then, we numerically investigate the time evolution equation (the gradient flow of the free energy), and see that the free energy has many local minimizers and the systems have some kind of sensitivity with respect to initial data.

MSC:

35Q72	Other PDE from mechanics (MSC2000)
82D60	Statistical mechanical studies of polymers
74N20	Dynamics of phase boundaries in solids

Keywords:

variational problem; long-range interaction; Gibbs free energy; phase separation; asymptotic expansion; local minimizers; sensitivity

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