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Exact Potts model partition function on strips of the triangular lattice. (English)

Zbl 1052.82513
Physica A 286, No. 1-2, 189-238 (2000).

Summary: In this paper we present exact calculations of the partition function Z of the q-state Potts model and its generalization to real q, for arbitrary temperature on n-vertex strip graphs, of width $L_y = 2$ and arbitrary length, of the triangular lattice with free, cyclic, and Möbius longitudinal boundary conditions. These partition functions are equivalent to Tutte-Whitney polynomials for these graphs. The free energy is calculated exactly for the infinite-length limit of the graphs, and the thermodynamics is discussed. Considering the full generalization to arbitrary complex q and temperature, we determine the singular locus B in the corresponding $\mathbb{C}^2$ space, arising as the accumulation set of partition function zeros as $n \to \infty$. In particular, we study the connection with the $T = 0$ limit of the Potts antiferromagnet where B reduces to the accumulation set of chromatic zeros. Comparisons are made with our previous exact calculation of Potts model partition functions for the corresponding strips of the square lattice. Our present calculations yield, as special cases, several quantities of graph-theoretic interest.

MSC:

82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics

Keywords:

specific heat; partition function; triangular lattice strips; q-state Potts model; vertex strip graphs; free boundary conditions; cyclic boundary conditions; Möbius longitudinal boundary conditions; Tutte-Whitney polynomials

Full Text: DOI arXiv

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