Lambert, A.

Completely asymmetric Lévy processes confined in a finite interval. (English) Zbl 0970.60055

Ann. Inst. Henri Poincaré, Probab. Stat. 36, No. 2, 251-274 (2000).

Let $[0,a]$ be a finite interval and consider a Lévy process X_t which has only negative jumps (i.e., it is completely asymmetric) and admits transition densities. Write ψ for the characteristic (Laplace) exponent of X_t and denote by T the first exit time from $[0,a]$. The two-sided exit problem for such a process was studied by *J. Bertoin* [Bull. Lond. Math. Soc. 28, No. 5, 514-520 (1996; Zbl 0863.60068) and Ann. Appl. Probab. 7, No. 1, 156-169 (1997; Zbl 0880.60077)]. Building on these earlier results, the author considers the problem to find the distribution $\mathbb{P}_x^\updownarrow$, $x\in[0,a]$, under which the process X_t stays in $[0,a]$. This is achieved by showing that the limit $\lim_{t\to\infty}\mathbb{P}_x(\Lambda\mid T>t=\mathbb{P}_x^\updownarrow(\Lambda)$ exists and defines a new probability measure under which $(X_t,\mathbb{P}_x^\updownarrow)$ is a Feller process with values in $[0,a]$. It turns out that $\mathbb{P}_x^\updownarrow$ can be realized as a Doob h-transform of the original measure $\mathbb{P}_x$ with respect to the $\mathbb{P}_x$-martingale (under the original filtration)

$$D_t=e^{\rho t}1_{\{t<T\}}\frac{W^{(-\rho)}(X_t)}{W^{(-\rho)}(x)}.$$

In this formula, $W^{(q)}(x)$ is the scale function of the process X_t killed at the constant rate q,

$$\int_0^\infty e^{-\lambda x}W^{(q)}(x)dx=\frac{1}{\psi(\lambda)-q},$$

and $\rho=\rho(a)$ is the first zero of $q\mapsto W^{(-q)}(a)$. Several properties of $\mathbb{P}^\updownarrow$ and of X_t under $\mathbb{P}^\updownarrow$ are established, e.g., an explicit representation of the resolvent kernels in terms of the scale function is given and the two-sided exit problem within $[0,a]$ is solved.

Moreover, the author studies some elements of fluctuation theory of $(X_t,\mathbb{P}^\updownarrow)$, in particular the excursion measure (away from a point $x\in[0,a]$) of the confined process which is expressed in terms of the excursion measure of the original process. This is then used to study local times L_t^x (and their inverses) of the confined process, and the value of the almost sure limit $\lim_{t\to\infty}(L_t^x/t)$ is explicitly found. If X_t is of unbounded variation, $t(a-\sup_{s\le t}X_s)$ converges as $t\to\infty$ in distribution to an exponential random variable with parameter $|\rho'(a)|$; a simple integral criterion is provided for convergence or divergence of the upper and lower limits of $(a-\sup_{s\le t}X_s)/f(t)$, $t\to\infty$. The proofs use variants of techniques for unconfined processes [cf. the monograph "Lévy processes" by *J. Bertoin* (1996; Zbl 0861.60003)], but they rely crucially on the explicit form of the law and excursion measure of the confined process.

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MSC:

60G51	Processes with independent increments; Lévy processes
60G17	Sample path properties
60J45	Probabilistic potential theory
60G55	Point processes (e.g., Poisson, Cox, Hawkes processes)

Keywords:
Lévy process; two-sided exit problem; conditional law; h-transform; Mittag-Leffler function; excursion measure

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