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Asymptotics of a dynamic random walk in a random scenery. I: Law of large numbers. (English) Zbl 0969.60045

Ann. Inst. Henri Poincaré, Probab. Stat. 36, No. 2, 127-151 (2000).

Define a non-homogeneous Markov chain $(S_n)_{n\in\mathbb{N}_0}$ on $\mathbb{Z}$ as follows. Fix some x in the d-dimensional torus $\mathbb{T}^d$ and a vector $\alpha \in \mathbb{R}^d$ having irrational components and some function $f : \mathbb{T}^d \to [0,1]$. Put $S_0 = 0$, and the steps $(S_i - S_{i-1})$ of the walk are assumed to be independent and to assume the value 1 with probability $f({\tau_\alpha}^i x)$ and the value -1 otherwise, where $\tau_\alpha : \mathbb{T}^d \to \mathbb{T}^d$ is the rotation by α on the torus. Furthermore, let $(\xi(z))_{z\in\mathbb{Z}}$ be an i.i.d. sequence of real random variables, acting as a random scenery, and define a random walk in random scenery by $Z_n = \sum_{i=0}^{n} \xi(S_i)$. The main goal of the paper is the proof for the facts that $(S_n)_n$ is recurrent on its moving average (i.e., $P(\limsup_{n\to\infty}\{|S_n - ES_n| < \varepsilon\}) = 1$ for any $\varepsilon > 0$), and that $(Z_n)_n$ satisfies a weak law of large numbers.

The results are formulated more precisely as follows. Let f be of bounded variation in the sense of Hardy and Krause, and assume that $a = 4\int_{\mathbb{T}^d} f(t)(1-f(t))dt$ is positive and that $\int_{\mathbb{T}^d} f(t)dt = 1/2$. (These two conditions ensure, via an ergodic theorem, that the limiting drift of the walk is zero and that the limiting variance is positive.) Then $(S_n)_n$ is recurrent on its moving average, and (under some technical additional assumption) $P(S_{2n} = 0) \sim (a\pi n)^{-1/2}$ and $Z_n/n \to 0$ in probability. Furthermore, in the case that the components of α are rational, some natural sufficient conditions are given such that the recurrence of $(S_n)_n$ can be decided and such that $Z_n/n \to 0$ almost surely.

Reviewer: W.König (Berlin)

MSC:

60G50 Sums of independent random variables; random walks

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random walk; random scenery; continued fractions; Denjoy-Koksma's inequality; low discrepancy sequences

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