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New high-resolution central schemes for nonlinear conservation laws and convection-diffusion equations. (English) Zbl 0987.65085

J. Comput. Phys. 160, No. 1, 241-282 (2000).

The authors first review some central difference schemes and then propose a new fully discrete second order central difference scheme for systems of one dimensional hyperbolic conservation laws. This scheme has the merit that its numerical viscosity is of order $\mathcal{O}(\Delta x)^{2r-1})$ which is independent of $1\Delta t$. (The parameter r has never been defined in the paper, but it is presumably the order of the scheme.) Furthermore, this scheme becomes semi-discrete when $\Delta t \to 0$. In this case the scheme satisfies the scalar total-variation diminishing property. Extensions of the semi-discrete scheme to convection-diffusion equations and to multi-dimensions are also discussed. Fully discrete schemes can also be obtained by applying Runge-Kutta methods to the semi-discrete formula. A number of numerical examples is solved by the method to justify the theoretical results obtained.

Reviewer: Song Wang (Nedlands)

MSC:

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
35L65 Hyperbolic conservation laws
65M20 Method of lines for initial value and initial-boundary value problems involving PDEs
65M15 Error bounds for initial value and initial-boundary value problems involving PDEs

Keywords:

hyperbolic conservation laws; multidimensional systems; degenerate diffusion; central difference schemes; non-oscillatory time differencing; total-variation diminishing property; semi-discrete scheme; convection-diffusion equations; Runge-Kutta methods; numerical examples

Software:

HLLE

Full Text: DOI

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