**Adolphson, Alan**; **Sperber, Steven**

**Dwork cohomology, de Rham cohomology, and hypergeometric functions.** (English)

Zbl 0980.14017
Am. J. Math. 122, No. 2, 319-348 (2000).

Let $S$ be a smooth affine $\mathbb{C}$-scheme, and let $X = \operatorname{Spec}(A)$ be an affine $S$-scheme. Consider a projective $\mathcal{O}_X$-module $\mathcal{E}$ of finite rank, and an integrable $\mathbb{C}$-connection $\nabla : \mathcal{E} \to \Omega^1_{X/\mathbb{C}} \otimes_{\mathcal{O}_X} \mathcal{E}$. Let $f_1, \ldots, f_r \in A$ be a regular sequence defining the smooth complete intersection $Y = \operatorname{Spec}(A/(f_1, \ldots, f_r))$, and let $j : Y \to X$ be the inclusion of $S$-schemes. Set $\mathbb{A}^r_X = \operatorname{Spec}(A[T_1, \ldots, T_r])$ so that $\pi : \mathbb{A}^r_X \to X$ is the projection. It is proved that for any $n \in \mathbb{N}$ there is an isomorphism of $\mathcal{O}_S$-modules with $\mathbb{C}$-connection: $H^n_{DR}(Y/S, (j^*(\mathcal{E}), \nabla_Y)) \cong H^{n+2r}_{DR}(\mathbb{A}^r_X/S, (\pi^*(\mathcal{E}), \nabla_F))$, where $\nabla_Y$ is the pullback of $\nabla$ to a connection on $j^*(\mathcal{E})$, $F = T_1 f_1 + \ldots + T_r f_r$, and $\nabla_F$ is the corresponding twisted connection. The cohomology groups on the right hand side can be interpreted as analogues of Dwork cohomology groups in the case of characteristic zero. This result can be considered as a generalization to the complete intersections case of a result by *N. Katz* [Publ. Math., Inst. Hautes Étud. Sci. 39, 175-232 (1970; Zbl 0221.14007)] which states that for smooth projective hypersurfaces the Dwork cohomology and the primitive part of the de Rham cohomology coincide. As an application the authors show how one can compute the Picard-Fuchs equations for smooth complete intersections.

Reviewer: Aleksandr G.Aleksandrov (Moskva)

**MSC:**

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| 14F40 | de Rham cohomology and algebraic geometry |
| 14F43 | Other algebro-geometric (co)homologies (e.g., intersection, equivariant, Lawson, Deligne (co)homologies) |
| 14D05 | Structure of families (Picard-Lefschetz, monodromy, etc.) |
| 53C05 | Connections, general theory |



**Keywords:**

integrable connection; Gauss-Manin connection; complete intersection; de Rham cohomology; Dwork cohomology; hypergeometric equations; Picard-Fuchs equations

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