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The $\mathcal{A}$-hypergeometric system associated with a monomial curve. (English) Zbl 0952.33009

Duke Math. J. 99, No. 2, 179-207 (1999).

Let $\mathcal{A}$ be a spanning subset of $\mathbb{Z}^{n+1}$ consisting of r elements, and let $\alpha \in \mathbb{C}^{n+1}$. In the late eighties Gel'fand, Kapranov and Zelevinskij associated with $\mathcal{A}$ and α a holonomic system of differential equations in $\mathbb{C}^r$, called the $\mathcal{A}$-hypergeometric system with exponent (or parameter) α. Its solutions are called the $\mathcal{A}$-hypergeometric functions with parameter α [see *I. M. Gel'fand*, *A. V. Zelevinskij* and *M. M. Kapranov*, Funct. Anal. Appl. 23, No. 2, 94-106 (1989; Zbl 0721.33006); Adv. Math. 84, No. 2, 255-271 (1990; Zbl 0741.33011)]. In the literature $\mathcal{A}$-hypergeometric systems are also called GKZ-systems. The paper under review studies the case of $\mathcal{A}$-hypergeometric systems associated with monomial curves, which corresponds to the case $n = 1$. All rational $\mathcal{A}$-hypergeometric functions with parameter α are shown to be Laurent polynomials. This property is proven by counterexample not to be true in the general case $n > 1$. The rational $\mathcal{A}$-hypergeometric functions with parameter $\alpha \in \mathbb{Z}^2$ are shown to span a space of dimension at most 2. The value 2 is attained if and only if the monomial curve is not arithmetically Cohen-Macaulay. For all values of α, the holonomic rank $r(\alpha)$ of the system is proven to satisfy the inequalities $d \leq r(\alpha) \leq d + 1$. Moreover $r(\alpha) = d + 1$ exactly for all $\alpha \in \mathbb{Z}^2$ for which the space of rational solutions has dimension 2. The inequalities for the holonomic rank have also been obtained using different methods by *M. Saito*, *B. Sturmfels* and *N. Takayama* [Gröbner deformations of hypergeometric differential equations. Springer-Verlag (2000; Zbl 0946.13021)].

Reviewer: A.Pasquale (Clausthal-Zellerfeld)

MSC:

33C70 Other hypergeometric functions and integrals in several variables
14D99 Families, fibrations in algebraic geometry
32G99 Deformations of analytic structures
33D15 Basic hypergeometric functions in one variable, ${}_r\phi_s$

Cited in **16** Documents

Keywords:

GKZ-systems; $\mathcal{A}$-hypergeometric function; $\mathcal{A}$-hypergeometric system

Full Text: DOI

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