Hunt, G. W.; **Lord, G. J.**; **Champneys, A. R.**

Homoclinic and heteroclinic orbits underlying the post-buckling of axially-compressed cylindrical shells. (English) Zbl 0958.74021

Comput. Methods Appl. Mech. Eng. 170, No. 3-4, 239-251 (1999).

Summary: A structural system with an unstable post-buckling response that subsequently restabilizes has the potential to exhibit homoclinic connections from the fundamental equilibrium state to itself over a range of loads, and heteroclinic connections between fundamental and periodic periodic equilibrium states over a different (smaller) range of loads. It is argued that such equilibrium configurations are important in the interpretation of observed behaviour, and govern the minimum possible post-buckling loads.

To illustrate this, the classical problem of a long thin axially-compressed cylindrical shell is revisited from three different perspectives: asymptotic conjecture, analogy with nonlinear dynamics, and numerical continuation analysis of a partial spectral decomposition of the underlying equilibrium equations. The nonlinear dynamics analogy demonstrates that the structure of the heteroclinic connections is more complicated than that indicated by the asymptotics: this is confirmed by the numerics. However, when the asymptotic portrayal is compared to the numerics, it turns out to be surprisingly accurate in its Maxwell-load prediction of the practically-significant first minimum to appear in the post-buckling regime.

MSC:

74G60 Bifurcation and buckling
74K25 Shells
37N15 Dynamical systems in solid mechanics

Keywords:

post-buckling response; homoclinic connections; heteroclinic connections; long thin axially-compressed cylindrical shell; nonlinear dynamics; numerical continuation analysis; spectral decomposition

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