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Finding rational points on bielliptic genus 2 curves. (English) Zbl 1029.11024

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Summary: The authors discuss a technique for trying to find all rational points on curves of the form $Y^2 = f_3X^6 + f_2X^4 + f_1X^2 + f_0$, where the sextic has nonzero discriminant. This is a bielliptic curve of genus 2. When the rank of the Jacobian is 0 or 1, Chabauty's theorem may be applied. However, they concentrate on the situation when the rank is at least 2. In this case, they derive an associated family of elliptic curves, defined over a number field $\mathbb{Q}(\alpha)$. If each of these elliptic curves has rank less than the degree of $\mathbb{Q}(\alpha) : \mathbb{Q}$, then they describe a Chabauty-like technique which may be applied to try to find all the points (x, y) defined over $\mathbb{Q}(\alpha)$ on the elliptic curves, for which $x \in \mathbb{Q}$. This in turn allows them to find all $\mathbb{Q}$-rational points on the original genus 2 curve. They apply this to give a solution to a problem of Diophantus (where the sextic in X is irreducible over $\mathbb{Q}$), which simplifies the recent solution given in the second author's Ph.D. thesis, Univ. California, Berkeley (1997). The authors also present two examples where the sextic in X is reducible over $\mathbb{Q}$.

MSC:

11G30 Curves of arbitrary genus or genus $\neq 1$ over global fields

14G05 Rational points

Keywords:

rational points; sextic; bielliptic curve of genus 2; Chabauty's theorem

Full Text: DOI