Kapovich, Ilya

A non-quasiconvexity embedding theorem for hyperbolic groups. (English) Zbl 0942.20026

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The author considers quasiconvex subgroups, and we recall the definition. A subset Y of a metric space X is said to be (ϵ)-quasiconvex if for every pair of points in Y, any geodesic segment joining them is contained in the ϵ-neighborhood of Y. Let G be now a hyperbolic group, equipped with a finite generating set, let K be the corresponding Cayley graph equipped with its word metric and let A be a subgroup of G. Then A is said to be a quasiconvex subgroup of G if it is quasiconvex as a subset of the metric space K.

The main result of this paper is the following Theorem A. If G is a not virtually cyclic torsion free hyperbolic group then there exists another word hyperbolic group G^* such that G is a subgroup of G^* but not quasiconvex in G^*. – As the author points out, examples of finitely generated subgroups of hyperbolic groups that are not quasiconvex were rare.

Theorem B. Let G be a torsion-free hyperbolic group and let Γ be a non-cyclic subgroup of G. Then, there exists a subgroup H of Γ such that H is a free group of rank two which is quasiconvex and malnormal in G (meaning that for any $g \in G - H$ we have $H \cap g^{-1}Hg = 1$). – The author discusses also a parallel between quasiconvexity and geometric finiteness for a group.

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MSC:

20F67 Hyperbolic groups and nonpositively curved groups
20F65 Geometric group theory
57M07 Topological methods in group theory
20E07 Subgroup theorems; subgroup growth

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