

**Garikipati, Krishna; Hughes, Thomas J. R.**

**A study of strain localization in a multiple scale framework. – The one-dimensional problem.**

(English) [Zbl 0961.74009](#)

Comput. Methods Appl. Mech. Eng. 159, No. 3-4, 193-222 (1998).

The authors investigate large strains localized in small regions. They split the displacement field into “coarse” and “fine” components. Using finite element formulations and linearized iterative procedure of Newton-Raphson type, the authors study softening problems in order to recover regularizing effects based on the dissipated energy. Rate-independent plasticity is treated by means of mean-sensitive Galerkin method. Then the authors describe formulations which include discrete models and crack band theory, give a nonlocal formulation based on weighted strains, analyse a multiple scale model characterized by a stiffness matrix, and determine the variations of tangent modulus. The conclusions show that the standard Galerkin method is unable to represent arbitrary fine scales, and that the regularizing technique provides coarse scale solutions free of pathological mesh dependence. Numerical simulations are performed for inviscid strain softening and viscoplastic strain localization. Plots show the displacement growth beginning from the onset of localization under loading, illustrate the invariance of discretized solutions and demonstrate the convergence of quadratures.

Reviewer: [M.Mișicu \(București\)](#)

**MSC:**

[74C20](#) Large-strain, rate-dependent theories of plasticity

[74C15](#) Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)

[74S05](#) Finite element methods applied to problems in solid mechanics

Cited in **1** Review  
Cited in **28** Documents

**Keywords:**

regularization technique; strain localization; splitting of displacement field; fine scale; large strains; iterative procedure of Newton-Raphson type; regularizing effects; mean-sensitive Galerkin method; crack band theory; multiple scale model; inviscid strain softening; viscoplastic strain localization

**Full Text:** [DOI](#)

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