

**Lin, Yanping; Liu, James H.**

**Stability of nonlocal diffusion equations.** (English) Zbl 0945.45004  
Dyn. Contin. Discrete Impulsive Syst. 5, No. 1-4, 53-66 (1999).

The partial integro-differential equation

$$u_t + Au + \int_0^t K(t-s)Bu(s)ds + f(u) = 0, \quad 0 < t \leq T,$$

is studied on a smooth domain  $\Omega$  with zero boundary conditions. The operator  $A$  is a strongly elliptic operator, so that the equation is parabolic in the absence of the integral term. The operator  $B$  is a second order differential operator with respect to  $x$ , and  $K$  is a scalar nonnegative kernel. A number of theorems is proved on exponential decay and time-discretization by backward Euler difference methods. In addition, the well-posedness and time-discretization of this problem are studied under a nonstandard non-local time weighted initial condition

$$u(x, 0) = \sum_{k=1}^M \beta_k(x)u(x, T_k) + \psi(x), \quad x \in \Omega,$$

where  $0 < T_1 < T_2 \cdots < T_M = T$ .

Reviewer: O.Staffans (Åbo)

**MSC:**

- 45K05 Integro-partial differential equations
- 45M10 Stability theory for integral equations
- 65R20 Numerical methods for integral equations

**Keywords:**

stability; nonlocal diffusion equations; partial integro-differential equation; backward Euler difference methods