Neukirch, Jürgen

Algebraic number theory. Transl. from the German by Norbert Schappacher. (English)
Zbl 0956.11021

Grundlehren der Mathematischen Wissenschaften. 322. Berlin: Springer. xvii, 571 p. (1999).

This is an English translation of a book, whose German original has been reviewed (Zbl 0747.11001). It brings in seven chapters a well-written introduction into modern number theory.

The first chapter presents the fundamental results including ideal theory (with Kummer's factorization theorem), Dirichlet's unit theorem, finiteness of the class-number and Hilbert's ramification theory. One finds here also a study of orders, which is welcome, as this topic is usually omitted in most textbooks. Also a link to algebraic geometry is provided: one-dimensional schemes are defined, as well as the Picard and Chow groups and a connection with the theory of function fields is sketched.

The second chapter brings valuation theory, including a study of Henselian fields and their extensions and in the next chapter this is applied to algebraic number fields. The theory of the discriminant and different is presented, Arakelov ideals and Arakelov class group are considered and a proof is given of an analogue of the Riemann-Roch theorem, based on A. Weil's definition of the genus for algebraic number fields. Then metrized modules over rings of integers are introduced and a formalism, which was introduced by A. Grothendieck in the case of algebraic varieties, is developed for these modules. After defining compactified Grothendieck groups (on which the tensor product induces a ring structure), the Chern character and Todd classes this construction culminates in the Grothendieck-Riemann-Roch theorem, relating, for a finite extension L/K of algebraic number fields, the Grothendieck groups corresponding to K and L. As the author states, this result "integrates completely the theory of algebraic integers into a general programme of algebraic geometry as a special case."

The next two chapters present local and global class field theory, modelled upon a previous work of the author ["Class Field Theory", Springer Verlag (1986; Zbl 0587.12001)] but including certain modifications and fresh examples.

The last chapter deals with zeta functions and L series. Contrary to most treatments of this topic the author does not use the approach based on harmonic analysis, but proceeds with a careful presentation of ideas of E. Hecke. This seems to be the first modern exposition of Hecke's method.

This book is a most welcome addition to the literature and will serve as a learning tool for years to come. The translator made a splendid job, preserving the lucid style of the original.

Reviewer: Władysław Narkiewicz (Wrocław)

MSC:

11Rxx	Algebraic number theory: global fields
11Sxx	Algebraic number theory: local and p-adic fields
11-02	Research exposition (monographs, survey articles) pertaining to number theory
11-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory

Keywords:

algebraic numbers; algebraic number theory; valuation theory; Henselian fields; Arakelov ideals; Arakelov class group; Riemann-Roch theorem; Chern character; Todd classes; Grothendieck-Riemann-Roch theorem; Grothendieck groups; local and global class field; zeta functions; L-series; Hecke's method; orders; Picard group; Chow group