Sznitman, Alain-Sol

Brownian motion, obstacles and random media. (English) Zbl 0973.60003

Springer Monographs in Mathematics. Berlin: Springer. xvi, 353 p. (1998).

This book is devoted to the survival of Brownian motion in $\mathbb{R}^d$ under the influence of soft (and hard) Poisson obstacles over long time intervals both in the quenched and annealed regimes. In a systematic, clear, and unified way, the book presents deep and exciting results, largely obtained by the author during the last decade. It is aimed to graduate students and researchers interested in recents progress in the theory of random media and related topics.

Part I (Chapters 1-3) contains background material on the Feynman-Kac formula, Dirichlet forms, potential theory, and eigenvalue estimates partially difficult to find in the iterature. It may also serve as a concrete introduction to nontrivial aspects of these subjects. Part II (Chapters 4-7) starts with a detailed description of the method of enlargement of obstacles developed by the author. This method allows for building a coarse-grained picture of the original Poisson cloud of obstacles which has much less combinatorial complexity, is easier to handle and allows uniform control on principal Dirichlet eigenvalues. Then a study of Lyapunov exponents follows characterizing crossings of Brownian motion among Poisson obstacles over long distances. Much attention is paid to the long-time behavior of the quenched path measure

$$Q_{t,\omega} = S_{t,\omega}^{-1} \exp\left\{ -\int_0^t V_\omega(Z_s) ds \right\} P_0,$$

where P_0 is the Wiener measure, Z denotes canonical Brownian motion, V_ω represents the random potential of Poisson clouds, and $S_{t,\omega}$ is the normalizing constant. As one of the culmination points, the author develops the mathematical foundations of the so-called pinning effect for the quenched measures $Q_{t,\omega}$: For a.e. realization V_ω, with overwhelming $Q_{t,\omega}$ probability as $t \to \infty$, Brownian motion soon hits a far region of small potential (determined by a random variational problem) and does not leave it up to time t. This is the geometric picture suggested by the localization theory of random Schrödinger operators. Finally, the author gives a short overview over related topics such as intermittency, large deviations, the confinement property, and the influence of a drift.

Reviewer: Jürgen Gärtner (Berlin)

MSC:

60-02 Research exposition (monographs, survey articles) pertaining to probability theory
60K40 Other physical applications of random processes
60J45 Probabilistic potential theory

Keywords:

Brownian motion; random media; Poisson obstacles; Brownian survival; enlargement of obstacles