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A domain embedding method for Dirichlet problems in arbitrary space dimension. (English)

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Author's abstract: An embedding method for the discretization of Dirichlet boundary value problems over general domains in arbitrary space dimension is proposed. The main advantage of the method lies in the use of Cartesian coordinates independent of the underlying domain. Error estimates and aspects of the numerical realization are considered. To obtain an efficient solver for the resulting linear system of equations an easy-to-use preconditioning is recommended and analyzed. A variety of numerical experiments illustrate and confirm the theoretical results.

Reviewer: [Th.Sonar \(Hamburg\)](#)

MSC:

- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
- [35J25](#) Boundary value problems for second-order elliptic equations
- [65N15](#) Error bounds for boundary value problems involving PDEs
- [65F10](#) Iterative numerical methods for linear systems
- [65F35](#) Numerical computation of matrix norms, conditioning, scaling

Cited in **1** Review
Cited in **2** Documents

Keywords:

[domain embedding method](#); [Dirichlet problems](#); [error estimates](#); [preconditioning](#); [numerical experiments](#)

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