El-Daou, M. K.; **Ortiz, E. L.**
The Tau method as an analytic tool in the discussion of equivalence results across numerical methods. (English) Zbl 0908.65072
Computing 60, No. 4, 365-376 (1998).

Authors' abstract: A Tau method approximate solution of a given differential equation defined on a compact $[a,b]$ is obtained by adding to the right-hand side of the equation a specific minimal polynomial perturbation term $H_n(x)$, which plays the role of a representation of zero in $[a,b]$ by elements of a given subspace of polynomials. Neither discretization or orthogonality are involved in this process of approximation. However, there are interesting relations between the Tau method and approximation methods based on the former techniques.

In this paper, we use equivalence results for collocation and the Tau method, contributed recently by the authors together with classical results in the literature, to identify precisely the perturbation term $H(x)$ which would generate a Tau method approximate solution, identical to that generated by some specific discrete methods over a given mesh $\Pi \in [a,b]$. Finally, we discuss a technique which solves the inverse problem, that is, to find a discrete perturbed Runge-Kutta scheme which would simulate a prescribed Tau method. We have chosen, as an example, a Tau method which recovers the same approximation as an orthogonal expanion method. In this way, we close the diagram defined by finite difference methods, collocation schemes, spectral techniques and the Tau method through a systematic use of the latter as an analytical tool.

Reviewer: Z.Jackiewicz (Tempe)

MSC:

65L60 Finite element, Rayleigh-Ritz, Galerkin and collocation methods for ordinary differential equations
65L05 Numerical methods for initial value problems
34A34 Nonlinear ordinary differential equations and systems, general theory

Keywords:

Tau method; collocation; finite difference methods; spectral methods

Software:

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