Mahavier, W. T.

A numerical method utilizing weighted Sobolev descent to solve singular differential equations. (English) Zbl 0908.65060

Nonlinear World 4, No. 4, 435-455 (1997).

Author's abstract: A numerical method is developed for solving singular differential equations using steepest descent based on weighted Sobolev gradients. The differential equation is cast as a least-squares problem yielding a functional representing the equation. A weighted Sobolev space is chosen which depends on both the functional and the boundary conditions, thus gradients associated with the functional take into account the singularity and the boundary conditions for the given equation. Results are presented for a variety of first- and second-order problems, including linear constrained, unconstrained, and partially constrained first-order problems, a nonlinear first-order problem with irregular singularity, and two singular second-order variational problems. Significant improvements are obtained by computing based on weighted Sobolev gradients rather than computing based on unweighted Sobolev gradients.

Reviewer: Z.Jackiewicz (Tempe)

MSC:

65L05	Numerical methods for initial value problems
34C05	Topological structure of integral curves, singular points, limit cycles of ordinary differential equations
34A34	Nonlinear ordinary differential equations and systems, general theory
34B15	Nonlinear boundary value problems for ordinary differential equations
65L10	Numerical solution of boundary value problems involving ordinary differential equations
65L12	Finite difference and finite volume methods for ordinary differential equations

Keywords:

finite differences; preconditioning; singular differential equations; steepest descent; weighted Sobolev gradients; least-squares problem