

Caffarelli, Luis A.; Kenig, Carlos E.

Gradient estimates for variable coefficient parabolic equations and singular perturbation problems. (English) [Zbl 0907.35026](#)

Am. J. Math. 120, No. 2, 391-439 (1998).

It is studied the regularity of spatial gradients of solutions to second order uniformly parabolic equations in divergence form, with bounded lower order terms and Dini continuous coefficients of the type

$$\operatorname{div} A(x, t) \nabla u - \partial_t u + b_i(x, t) \frac{\partial u}{\partial x_i}(x, t) + c(x, t) u(x) = f(x, t) + \operatorname{div} \vec{g}(x, t).$$

Uniform spatial Lipschitz estimates are established for some singular perturbation problems. It is used a monotonicity formula approach to be shown that Dini continuity of the coefficients at a given point yields boundedness of the gradient at the same point. It is obtained a modulus of continuity estimate up to the boundary for gradients of solutions with smooth Dirichlet data. A compactness argument and a judicious application of Harnack's principle allow to obtain a Hopf maximum principle for the considered class of equations.

Reviewer: [Lubomira Softova \(Sofia\)](#)

MSC:

- [35B45](#) A priori estimates in context of PDEs
- [35K20](#) Initial-boundary value problems for second-order parabolic equations
- [35B25](#) Singular perturbations in context of PDEs
- [35B50](#) Maximum principles in context of PDEs
- [35B65](#) Smoothness and regularity of solutions to PDEs

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Keywords:

[Dini continuous coefficients](#); [monotonicity formula approach](#); [smooth dirichlet data](#); [Hopf maximum principle](#)

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