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Cannon-Thurston maps for trees of hyperbolic metric spaces. (English) Zbl 0906.20023

J. Differ. Geom. 48, No. 1, 135-164 (1998).

If X is a hyperbolic space in the sense of Gromov, then ∂X denotes the Gromov boundary of X and $\widehat{X} = X \cup \partial X$ the Gromov compactification. If Y and X are two hyperbolic metric spaces and if $i\colon Y \to X$ is a proper embedding, then a Cannon-Thurston map $\widehat{i}\colon \widehat{Y} \to \widehat{X}$ is a continuous extension of i. It is easy to see that if such a continuous extension exists, then it is unique, and the problem is therefore the existence of such a map.

In this paper, the authors define the notion of a tree of hyperbolic spaces satisfying the quasi-isometrically embedded condition. This is given by a path metric space X, a simplicial tree T and an onto map $P\colon X \to T$ such that the inverse images of the edges and vertices of T by the map P satisfy several technical conditions. In particular, for each vertex v of T, the subset $X_v = P^{-1}(v)$ of X is a path connected rectifiable metric space which, equipped with the induced path metric, is a δ-hyperbolic metric space, with δ independent of the choice of the vertex v. Furthermore, the inclusions $X_v \to X$ are uniformly proper. The notion of a tree of hyperbolic spaces satisfying the quasi-isometrically embedded condition is closely related to a notion introduced by *M. Bestvina* and *M. Feighn* [in J. Differ. Geom. 35, No. 1, 85-102 (1992; Zbl 0724.57029)].

The main result of this paper is the following Theorem. Let (X, T) be a tree of hyperbolic metric spaces satisfying the quasi-isometrically embedded condition, let v be a vertex of T and let (X_v, d_v) denote the hyperbolic metric space corresponding to v. Then, if X is hyperbolic, there exists a Cannon-Thurston map for (X_v, X).

The authors give several corollaries of this theorem. For instance, they give a new proof of Thurston's Ending Lamination Conjecture for geometrically tame manifolds with freely indecomposable fundamental group and with a uniform lower bound on the injectivity radius. (The conjecture has been first proved by Y. Minsky.)

The authors prove also the following (also proved by *E. Klarreich*, in his Ph. D. thesis, SUNY Stony Brook, 1997): Theorem. Let Γ be a freely indecomposable Kleinian group such that the injectivity radius of H^3/Γ is uniformly bounded below by some $\epsilon > 0$. Then, there exists a continuous map from $\partial\Gamma$ to the limit set of Γ in $S^2 = \partial H^3$. – Other applications concern graphs of hyperbolic groups, and the problem of local connectivity of limit sets of Kleinian groups. Finally, the authors describe examples where the existence of a Cannon-Thurston map is not known.

Reviewer: A.Papadopoulos (Strasbourg)

MSC:

20F65	Geometric group theory
57M50	General geometric structures on low-dimensional manifolds
30F40	Kleinian groups (aspects of compact Riemann surfaces and uniformization)
53C23	Global geometric and topological methods (à la Gromov); differential geometric analysis on metric spaces

Keywords:

Gromov hyperbolic spaces; Cannon-Thurston maps; hyperbolic groups; graphs of groups; trees of hyperbolic metric spaces; Gromov compactifications; quasi-isometrically embedded condition; freely indecomposable Kleinian groups

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