Bowditch, Brian H.

A topological characterisation of hyperbolic groups. (English) Zbl 0906.20022

J. Am. Math. Soc. 11, No. 3, 643-667 (1998).

The main result in this paper is the following Theorem: Let M be a perfect metrizable compact Hausdorff space and let Γ be a group of homeomorphisms of M, such that the induced action on the space of distinct triples in M (that is, the space $M \times M \times M$ minus the large diagonal) is properly discontinuous and cocompact. Then, Γ is a hyperbolic group. Furthermore, there exists a Γ-equivariant homeomorphism of M onto $\partial\Gamma$. This result was conjectured by Gromov.

Along the way, the author gives a characterization of hyperbolic groups as being convergence groups (in the sense defined by *F. W. Gehring* and *G. J. Martin* [in Proc. Lond. Math. Soc., III. Ser. 55, 331-358 (1987; Zbl 0628.30027)]) for which every point is a conical limit point. As the author says, results having the same flavour have been obtained by *P. Tukia* [in J. Reine Angew. Math. 501, 71-98 (1998)].

The ingredients used in this paper include the notion of quasiconformal structure, which can be defined in terms of crossratios, or of annulus systems, and one of the main steps in the paper is the fact that a suitable quasiconformal structure on a set gives rise to a hyperbolic quasimetric on the set of distinct triples (a quasimetric being defined as a metric except that the triangle inequality is relaxed by the introduction of an additive constant).

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MSC:

20F65	Geometric group theory
57M07	Topological methods in group theory
57S05	Topological properties of groups of homeomorphisms or diffeomorphisms

Keywords:

hyperbolic groups; convergence groups; boundaries; crossratios; trees; quasimetrics; quasiconformal structures; homeomorphism groups

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