Kobayashi, Shoshichi

Hyperbolic complex spaces. (English) Zbl 0917.32019

Grundlehren der Mathematischen Wissenschaften. 318. Berlin: Springer. xiii, 471 p. (1998).

A Riemann surface is called hyperbolic if its universal covering is the unit disc D with a natural metric of constant curvature -1. An extension of the notion of hyperbolicity to complex spaces, of any dimension ought to be such that the hyperbolic case is the generic case.

Kobayashi's approach starts with intrinsic pseudo-distances. The Carathéodory pseudo-distance of two points p, q of a complex space X reads

$$c(p,q) = \sup \rho\big(f(p), f(q)\big),$$

where the supremum is taken over all holomorphic maps $f\colon X \to D$ and ρ denotes the Poincaré distance in D. Carathéodory introduced this c in 1926 for a domain in $\mathbb{C}^n$. Kobayashi in 1967 defined another pseudo-distance d which is in a sense dual to c, namely a supremum is taken over all holomorphic maps $f\colon D \to X$. More precisely, d is the largest pseudo-distance such that all holomorphic maps $f\colon (D,\rho) \to (X,d)$ are distance-decreasing. This is motivated by Ahlfors' generalization of the Schwarz lemma. Now a complex space X is called hyperbolic if its Kobayashi pseudo-distance d is actually a distance.

The concept proved to be very fruitful. In the three decades since 1967 the subject of hyperbolic complex spaces has grown to a big industry with numerous articles and several books, including a little monograph (1970) and a long survey article (1976) by Kobayashi himself. The present book, which soon advances to a standard reading in complex geometry, gives a comprehensive account on intrinsic distances, hyperbolic complex spaces, and holomorphic maps between such spaces. The methods are mainly differential-geometric ones.

The eight chapters are headlined as follows. 1. Distance geometry. 2. Schwarz Lemma and negative curvature. 3. Intrinsic distances. 4. Intrinsic distances for domains. 5. Holomorphic maps into hyperbolic spaces. 6. Extension and finiteness theorems. 7. Manifolds of general type. 8. Value distributions.

The first two chapters provide needed facts of metric space theory and complex function theory. Chapter 3 is the core of the book; it develops the fundamental concepts. The other chapters discuss several topics which are connected with hyperbolicity, hyperbolic imbedding etc. The reference list with more than 600 entries is in its completeness of greatest value for all researchers in the field.

Reviewer: R.Schimming (Greifswald)

MSC:

32Q45	Hyperbolic and Kobayashi hyperbolic manifolds
32F45	Invariant metrics and pseudodistances in several complex variables
32-02	Research exposition (monographs, survey articles) pertaining to several complex variables and analytic spaces
53-02	Research exposition (monographs, survey articles) pertaining to differential geometry
32C15	Complex spaces
53C60	Global differential geometry of Finsler spaces and generalizations (areal metrics)

Keywords:

intrinsic distances; hyperbolic complex spaces; holomorphic maps