

Adleman, Leonard M.; Huang, Ming-Deh A.

Counting rational points on curves and abelian varieties over finite fields. (English)

[Zbl 0898.11045](#)

Cohen, Henri (ed.), Algorithmic number theory. Second international symposium, ANTS-II, Talence, France, May 18-23, 1996. Proceedings. Berlin: Springer. Lect. Notes Comput. Sci. 1122, 1-16 (1996).

Let A be an abelian variety of dimension g over a finite field \mathbb{F}_q . Suppose that A is given as a closed subvariety of projective n -space. The authors exhibit a deterministic algorithm that computes the characteristic polynomial of the Frobenius endomorphism of A that runs in time $O((\log q)^c)$, where c is a polynomial expression in g as well as n . This improves upon an earlier result of *J. Pila* [Math. Comput. 55, 745-763 (1990; [Zbl 0724.11070](#))], who obtained a similar result but with the constant c depending exponentially on n .

By applying this to the Jacobian varieties of curves X over \mathbb{F}_q , one obtains a deterministic algorithm to count the number of \mathbb{F}_q -rational points of X that runs in time $O((\log q)^c)$, where c is a polynomial expression in n , as well as the genus g of X . In the special case of hyperelliptic curves of genus g , the authors show that the number of \mathbb{F}_q -rational points on X can be counted deterministically in time $(\log q)^{O(g^6)}$. This case is of interest in view of the primality test described by the authors in their monograph [Primality testing and abelian varieties over finite fields, Lect. Notes Math. 1512 (Springer-Verlag, 1992; [Zbl 0744.11065](#))].

For the entire collection see [[Zbl 0852.00023](#)].

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MSC:

- [11Y16](#) Number-theoretic algorithms; complexity
- [11G25](#) Varieties over finite and local fields
- [11G20](#) Curves over finite and local fields
- [14K15](#) Arithmetic ground fields for abelian varieties
- [14G05](#) Rational points
- [14G15](#) Finite ground fields in algebraic geometry
- [14Q05](#) Computational aspects of algebraic curves
- [14Q15](#) Computational aspects of higher-dimensional varieties
- [14H25](#) Arithmetic ground fields for curves

Cited in **5** Documents

Keywords:

abelian variety over a finite field; deterministic algorithm; characteristic polynomial of the Frobenius endomorphism; Jacobian varieties; rational points; hyperelliptic curves; primality test; complexity analysis