

Bergamasco, Adalberto P.; Zuffi, Edna M.

On global hypoellipticity on the torus. (English) Zbl 0886.35048

Tsukuba J. Math. 21, No. 2, 319-327 (1997).

Our main purpose is to study global hypoellipticity for a class of pseudodifferential operators on the n -torus, T^n , $n \geq 2$, of the form

$$P = p(D_1^2) + e^{imx_1} + ae^{-imx_1},$$

where $a = \pm 1$, $m \in \mathbb{N}$, $D_1 = (1/i)(\partial/\partial x_1)$, and p is a classical symbol satisfying the additional conditions

$$p(0) = 0; \quad |p(1)| \geq 1; \quad |p(t^2)| > 2, \quad t \in \mathbb{N}, \quad t \geq 2.$$

We recall that an operator P is said to be globally hypoelliptic (GH) on T^n if the properties $u \in \mathcal{D}'(T^n)$ and $Pu \in C^\infty(T^n)$ imply $u \in C^\infty(T^n)$. We present a necessary and sufficient condition for the operator to be (GH). Our examples show, in particular, that in the case when $p(t) = \lambda t^2$, $1 < \lambda < 2$, the situation $m > 1$ is different from the case $m = 1$, namely, when $m > 1$, the operator may fail to be (GH).

MSC:

35H10 Hypoelliptic equations

35S05 Pseudodifferential operators as generalizations of partial differential operators

Keywords:

necessary and sufficient condition for global hypoellipticity

Full Text: [DOI](#)