Le Calvez, Patrice; Yoccoz, Jean-Christophe

An index theorem for the homeomorphisms of the plane near a fixed point. (Un théorème d'indice pour les homéomorphismes du plan au voisinage d'un point fixe.) (French)

Zbl 0895.58032
Ann. Math. (2) 146, No. 2, 241-293 (1997).

This investigation was motivated by the following question: does the infinite annulus, $\mathbb{R}\times(\mathbb{R}/\mathbb{Z})$, admit a minimal homeomorphism? The authors compute the index, $i(f^k,z)$, of a sequence of k-th interates, $k=1,2,\dots$, of a local homeomorphism of $\mathbb{R}^2$ at a fixed point z which forms a locally maximal invariant set and is neither a sink nor a source. For suitably chosen integers $q,r\geq 1$ it satisfies the following condition:

$$i(f^k,z)=\begin{cases}1-rq, & \text{if } k \text{ is a multiple of } q,\\ 1, & \text{otherwise.}\end{cases}$$

Cyclically ordered sets play an important technical role in the proofs.

From the above result the authors deduce that the two-sphere $\mathbb{S}^2$, punctured finitely many times, admits no minimal homeomorphism. This important and definitive result includes, as particular cases, old results on the non-existence of minimal homeomorphisms on $\mathbb{S}^2$ and on $\mathbb{R}^2$, as well as the recent result by *M. Handel* [Ergodic Theory Dyn. Syst. 12, No. 1, 75-83 (1992; Zbl 0769.58037)], stating that $\mathbb{S}^2$ with ≥ 3 punctures admits no minimal homeomorphism. Thus, the question at the beginning of this review is answered negatively.

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MSC:

37B99	Topological dynamics
37C80	Symmetries, equivariant dynamical systems (MSC2010)
54H20	Topological dynamics (MSC2010)

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minimal homeomorphism; punctured sphere; infinite annulus; fixed point; index; cyclically ordered sets; rotation number

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