Neuberger, John W.

Sobolev gradients and differential equations. (English) Zbl 0935.35002

Lecture Notes in Mathematics. 1670. Berlin: Springer. viii, 149 p. (1997).

The author studies gradient methods to find zeros or critical points of a real-valued function Φ defined in general on an infinite dimensional space, mainly a Hilbert space H. Special emphasis is given to the case that Φ is related to solving a boundary value problem for an ordinary or partial differential equation (or a finite dimensional analogue of it), where then H is taken to be a suitable chosen Sobolev space.

The book is not organized to present a step by step developed theory of the method. Instead in its 18 chapters relevant aspects of gradient methods are presented and interesting applications are given. For example, the need of accelerating the convergence by preconditioning is pointed out (although this terminology is not explicitely used in the book). Other topics are how to integrate boundary conditions into the definition of Φ and H by using projected gradients and Newton's method, all in the general context of Sobolev gradients. Applications include the Ginzburg-Landau equations of superconductivity, minimal surfaces, flow problem and conservation type equations. Numerical calculations and graphical representations of the results illustrate the preformance of the methods.

Reviewer: R.D.Grigorieff (Berlin)

MSC:

35-02	Research exposition (monographs, survey articles) pertaining to partial differential equations
65N99	Numerical methods for partial differential equations, boundary value problems
35A15	Variational methods applied to PDEs
35A35	Theoretical approximation in context of PDEs
65J15	Numerical solutions to equations with nonlinear operators (do not use 65Hxx)

Keywords:

gradient method; steepest descent; Newton method; projected gradients; Ginzburg-Landau equation; minimal surfaces; accelerating the convergence by preconditioning