**Borodin, O. V.**; **Kostochka, A. V.**; **Woodall, D. R.**

**List edge and list total colourings of multigraphs.** (English) Zbl 0876.05032

J. Comb. Theory, Ser. B 71, No. 2, 184-204 (1997).

This paper exploits the remarkable new method of *F. Galvin* [J. Comb. Theory, Ser. B 63, No. 1, 153-158 (1995; Zbl 0826.05026)], who proved that the list edge chromatic number $\chi'_{\text{list}}(G)$ of a bipartite multigraph $G$ equals its edge chromatic number $\chi'(G)$. It is now proved here that if every edge $e = uw$ of a bipartite multigraph $G$ is assigned a list of at least $\max\{d(u), d(w)\}$ colours, then $G$ can be edge-coloured with each edge receiving a colour from its list. If every edge $e = uw$ in an arbitrary multigraph $G$ is assigned a list of at least $\max\{d(u), d(w)\} + \lfloor \frac{1}{2} \min\{d(u), d(w)\} \rfloor$ colours, then the same holds; in particular, if $G$ has maximum degree $\Delta = \Delta(G)$ then $\chi'_{\text{list}}(G) \leq \lfloor \frac{3}{2}\Delta \rfloor$. Sufficient conditions are given in terms of the maximum degree and maximum average degree of $G$ in order that $\chi'_{\text{list}}(G) = \Delta$ and $\chi''_{\text{list}}(G) = \Delta + 1$. Consequences are deduced for planar graphs in terms of their maximum degree and girth, and it is also proved that if $G$ is a simple planar graph and $\Delta \geq 12$ then $\chi'_{\text{list}}(G) = \Delta$ and $\chi''_{\text{list}}(G) = \Delta + 1$.

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