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Hadamard states, adiabatic vacua and the construction of physical states for scalar quantum fields on curved spacetime. (English) Zbl 0869.53053

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Once for a quantum field on a curved spacetime, the algebra A of observables has been constructed, the states can be defined as the normalized positive linear functionals on A. However, not every mathematical state is physically realizable. On Minkowski space it is understood that the physical states are exactly the elements of the "folium" (=quasiequivalence class) of the vacuum state. On a curved background, however, there is no vacuum state and a generally accepted method of how to determine the physical state does not exist. In the article under review, the author gives a detailed and comprehensive report on the partial results that have been achieved to overcome this problem. Throughout the Klein-Gordon equation is considered on a globally hyperbolic spacetime.

To keep the article self-contained the author begins with a chapter on mathematical preliminaries in which he reviews some basic facts about pseudodifferential operators, wavefront sets of distributions, and parametrices of the Klein-Gordon operator. In the main part of the article the fundamental notion of Hadamard states is introduced and several criteria are given for a state to be locally or globally Hadamard. It is argued that the quasiequivalence class of states generated by the Hadamard states is a good candidate for the physically realizable states. Here a result of *R. Verch* [Commun. Math. Phys. 160, 507-536 (1994; Zbl 0790.53077)] is essential who showed that any two Hadamard states of a Klein-Gordon field on a globally hyperbolic spacetime are quasiequivalent. Based on this discussion of Hadamard states it is proved (i) that ground- and KMS-states on static spacetimes are Hadamard states provided that the Lorentzian length of the Killing vector field is bounded from below by a positive constant, and (ii) that adiabatic vacuum states on Robertson-Walker spacetimes are Hadamard states. Moreover, a counterexample is presented which shows that neither the method of "Hamiltonian diagonalization" nor the construction of "energy states" necessarily yields a state in the folium of Hadamard states. Finally, a general method of how to construct Hadamard states on an arbitrary globally hyperbolic spacetime is given. This method is based on a local factorization of the wave operator with the help of pseudodifferential operators.

The article is carefully written and clearly organized. Everyone who is interested to study the details of this highly technical but physically interesting subject will find this report most useful.

Reviewer: V.Perlick (Berlin)

MSC:

53Z05 Applications of differential geometry to physics
81T20 Quantum field theory on curved space or space-time backgrounds
83C47 Methods of quantum field theory in general relativity and gravitational theory

Keywords:

Klein-Gordon equation; globally hyperbolic spacetime; Hadamard states

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