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Non-negative Feynman-Kac kernels in Schrödinger's interpolation problem. (English)

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Summary: The local formulations of the Markovian interpolating dynamics, which is constrained by the prescribed input-output statistics data, usually utilize strictly positive Feynman-Kac kernels. This implies that the related Markov diffusion processes admit vanishing probability densities only at the boundaries of the spatial volume confining the process. We discuss an extension of the framework to encompass singular potentials and associated nonnegative Feynman-Kac-type kernels. It allows us to deal with a class of continuous interpolations admitted by general nonnegative solutions of the Schrödinger boundary data problem. The resulting nonstationary stochastic processes are capable of both developing and destroying nodes (zeros) of probability densities in the course of their evolution, also away from the spatial boundaries. This observation conforms with the general mathematical theory (due to M. Nagasawa and R. Aebi) that is based on the notion of multiplicative functionals, extending in turn the well-known Doob's h-transformation technique. In view of emphasizing the role of the theory of nonnegative solutions of parabolic partial differential equations and the link with "Wiener exclusion" techniques used to evaluate certain Wiener functionals, we give an alternative insight into the issue, that opens a transparent route towards applications.

MSC:

35K10 Second-order parabolic equations
60J70 Applications of Brownian motions and diffusion theory (population genetics, absorption problems, etc.)

Keywords:

Markovian interpolating dynamics; Schrödinger boundary data problem; Wiener functionals

Full Text: DOI

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