Giudici, Reinaldo E.; **Lima de Sá, Eduardo**
Chromatic uniqueness of certain bipartite graphs. (English) Zbl 0862.05045
Congr. Numerantium 76, 69-75 (1990).

All graphs considered here are simple graphs. The chromatic polynomial of a graph X will be denoted by $P_X(\lambda)$. Two graphs X and Y are chromatically equivalent (or χ-equivalent) if they have the same chromatic polynomial. A graph X is chromatically unique (χ-unique) if X is isomorphic to every graph which is χ-equivalent to X.

We shall be concerned with bipartite graphs, i.e., graphs whose vertex set can be partitioned into two subsets H_1, H_2 such that every edge of the graph joints H_1 with H_2. We will denote by $K_{m,n}$ the complete bipartite graph for which H_1 has m vertices and H_2 has n and, in general will assume $m \leq n$. We prove that if G is obtained from $K_{m,m}$ or $K_{m,m+1}$ by deleting d disjoint edges, $0 \leq d \leq m$ and $m > 2$, then G is chromatically unique; the cases $d = 0$ and $d = 1$ were known. The case $d = 2$ gives a partial answer to Problem 12 stated in [*K. M. Koh* and *K. L. Teo*, Graphs Comb. 6, No. 3, 259-285 (1990; Zbl 0727.05023)], of determining the χ-uniqueness of graphs obtained from the complete $K_{m,n}$ graph by removing two of its edges; in fact, we prove that a graph obtained in this way from $K_{m,m}$ or $K_{m,m+1}$ is chromatically unique. We also prove that for each $k \geq 2$, the graphs obtained from $K_{m,m+k}$ by removing any two of its edges are χ-unique if m is large enough.

MSC:

05C15 Coloring of graphs and hypergraphs

Keywords:

chromatic polynomial; bipartite graphs