Garbaczewski, Piotr; **Olkiewicz, Robert**

Feynman-Kac kernels in Markovian representations of the Schrödinger interpolating dynamics. (English) Zbl 0869.60101

J. Math. Phys. 37, No. 2, 732-751 (1996).

Summary: Probabilistic solutions of the so-called Schrödinger boundary data problem provide for a unique Markovian interpolation between any two strictly positive probability densities designed to form the input-output statistics data for the process taking place in a finite-time interval. The key issue is to select the jointly continuous in all variables positive Feynman-Kac kernel, appropriate for the phenomenological (physical) situation. We extend the existing formulations of the problem to cases when the kernel is not a fundamental solution of a parabolic equation, and prove the existence of a continuous Markovian interpolation in this case. Next, we analyze the compatibility of this stochastic evolution with the original parabolic dynamics, which is assumed to be governed by the temporally adjoint pair of (parabolic) partial differential equations, and prove that the pertinent random motion is a diffusion process. In particular, in conjunction with Born's statistical interpretation postulate in quantum theory, we consider stochastic processes which are compatible with the Schrödinger picture quantum evolution.

MSC:

60K40 Other physical applications of random processes
82C31 Stochastic methods (Fokker-Planck, Langevin, etc.) applied to problems in time-dependent statistical mechanics

Keywords:

Schrödinger boundary data problem; Feynman-Kac kernel; statistical interpretation postulate in quantum theory; quantum evolution

Full Text: DOI arXiv

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