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Analysis and convergence of a covolume method for the generalized Stokes problem. (English)

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Summary: We introduce a covolume or MAC-like method for approximating the generalized Stokes problem. Two grids are needed in the discretization; a triangular one for the continuity equation and a quadrilateral one for the momentum equation. The velocity is approximated using nonconforming piecewise linears and the pressure piecewise constants. Error in the L^2 norm for the pressure and error in a mesh dependent H^1 norm as well as in the L^2 norm for the velocity are shown to be of first order, provided that the exact velocity is in H^2 and the true pressure in H^1. We also introduce the concept of a network model into the discretized linear system so that an efficient pressure-recovering technique can be used to simplify a great deal the computational work involved in the augmented Lagrangian method. Given is a very general decomposition condition under which this technique is applicable to other fluid problems that can be formulated as a saddle-point problem.

MSC:

65N15 Error bounds for boundary value problems involving PDEs
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
76D07 Stokes and related (Oseen, etc.) flows
35B45 A priori estimates in context of PDEs
35J50 Variational methods for elliptic systems

Keywords:

covolume methods; augmented Lagrangian method; nonconforming mixed finite element; network models

Full Text: DOI

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